Children's mathematical development and learning needs in perspective of teachers' use of dynamic math interviews

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Contents

Chapter 1

General introduction

Successful mathematical development is an important factor for daily functioning, self-reliance, and later career opportunities (Tout & Gal, 2015). In the first years of elementary school, children are expected to develop an understanding of numbers, counting, and basic arithmetic skills: the necessities for later mathematical development (Geary, 2004; Geary & Hoard, 2005). Half-way through elementary school, the complexity of mathematical problems increases and the focus of mathematics education shifts to advanced mathematics (e.g., fractions, percentages).

Large individual differences characterize children's mathematical development and have been found to be predicted by both general cognitive factors (e.g., reasoning, executive functioning, processing speed) and domain-specific skills (e.g., automatization of addition and subtraction up to 20) as well as beliefs and emotions (Bailey et al., 2014; Chinn, 2012; Cragg et al., 2017; Fuchs et al., 2016). Having an informed overview of children's math learning needs is thus crucial.

Dynamic math interviewing is an assessment approach involving an interactive, process-oriented, teacher-child dialogue and can provide insight into children's math learning needs (Allsopp et al., 2008). The aim of dynamic math interviewing is to identify the individual math learning needs of children, suitable forms of instruction, and the extent and type of support that is needed (Ginsburg, 1997, 2009; Van Luit, 2019; Wright et al., 2006). The conduct of dynamic math interviews requires specific knowledge and skills. A teacher professional development program that promotes the development of the necessary competencies could help teachers how to conduct such interviews (Heck et al., 2019).

Despite the widespread availability of research addressing the impact of child and teacher factors on mathematics achievement, relatively few studies have combined child and teacher factors to predict children's mathematical development. With regard to mathematical development, arithmetic fluency and mathematical problem-solving are generally not distinguished. And it has yet to be demonstrated that dynamic math interviewing is truly effective for the identification of math learning needs and improvement of mathematical teaching as a result. The aim of the present research was therefore to unravel the specific roles of child and teacher factors in children's mathematical development and to examine the extent to which dynamic math interviews facilitate the identification of math learning needs and promote teachers' mathematics teaching and children's mathematics learning.

Mathematical development

In the early years of elementary school, children develop an understanding of numbers, counting, and simple arithmetic (Geary, 2004). Children learn different solution methods, such as use of doubles, splitting, and deriving an answer from a known number combination. Considerable attention is paid to basic arithmetical skills and, over the years, accuracy improves and calculation processes speed up (Ostad, 2000). From about fourth grade (children aged 8-10 years), the transition is made to new domains of mathematics with increasingly abstract and complex problems (Geary, 2011).

Mathematics in elementary school involves various domains — such as number, number sense, operations, measurement, and ratios, which all require a conceptual understanding, procedural knowledge, and factual knowledge (National Research Council, 2001). Two particularly relevant aspects of mathematical development are arithmetic fluency and mathematical problem-solving (Fuchs et al., 2008).

Children are arithmetically fluent when they are able to add, subtract, multiply, and divide both quickly and accurately. And arithmetic fluency has been found to be essential for overall mathematics achievement (Fuchs et al., 2006). To become arithmetically fluent, conceptual understanding in combination with selection and application of appropriate strategies and extended practice is needed. In grade 4, it is to be expected that most children are arithmetically fluent.

Mathematical problem-solving is the ability to apply mathematical knowledge and skills to solve actual real-world or hypothetical problems using mathematical notation, text, and/or pictures (Polya, 1957). Mathematical problem-solving promotes analytical thinking and mathematical reasoning, which are skills that are obviously useful in later life and therefore required learning at school (Gravemeijer et al., 2017).

Arithmetic fluency and mathematical problem-solving can be distinguished but are also related to each other (Fuchs et al., 2008). Children who are not arithmetically fluent can have problems with the retrieval of basic arithmetic facts from long-term memory while trying to solve mathematical problems (Andersson, 2008; Duncan et al., 2007; Fuchs et al., 2016; Geary, 2011; Träff et al., 2020).

Child predictors of mathematical development

The mathematical development of children can be facilitated (and hindered) by general cognitive systems and domain-specific cognitive competencies, on the one hand, and by emotions and beliefs, on the other hand (Chinn, 2012; Fuchs et al., 2016; Lee, 2009). Hierarchical models of the role of cognitive systems in the development of mathematics (Cragg et al., 2017; Geary, 2004; Geary & Hoard, 2005) assume roles for the central executive control system, the visuospatial system, and the auditory-based phonological system (Baddeley, 2000). In addition, math self-efficacy and math self-concept along with math anxiety have been shown to be associated with mathematics achievement (Lee, 2009).

Cognitive predictors

The executive functions of visuospatial and verbal memory updating, inhibition, and shifting are all cognitive skills that are part of the central executive control system and thus provide crucial support for children's development of the domain-specific mathematical processes (i.e., conceptual understanding, factual knowledge, procedural skill) (Baddeley, 2000; Cragg et al., 2017; Cragg & Gilmore, 2014). Updating is the ability to monitor and manipulate task-relevant information held in mind; inhibition is the ability to suppress irrelevant information and inappropriate responses; and shifting is the capacity for flexible thinking and smoothly switching between tasks and strategies (Miyake et al., 2000). And all of the various executive functions have been shown to contribute to the individual differences observed in children's mathematical development (Bull & Scerif, 2001; Cragg & Gilmore, 2014).

Arithmetic fluency contributes to children's ability to use a variety of cognitive procedures appropriately and efficiently, which is essential for the development of more advanced mathematical problem-solving abilities (Gersten et al., 2005; Träff et al., 2020). Conceptual understanding in combination with the efficient application of strategies and extended practice are needed to speed up the calculation process. Automaticity facilitates working memory and allows children to further develop their mathematical problems-solving ability and acquire new mathematical concepts and skills (Geary, 2004). A lack of arithmetic fluency for basic mathematical facts can clearly hinder children's progress in mathematical problem-solving (Geary, 2011; Träff et al., 2020).

To solve a mathematical problem, children must be able to read the problem, distinguish relevant from irrelevant information, identify key words, devise a solution plan, determine underlying numerical relationships, select and apply required operations and algorithms, manipulate numbers, and — in doing all of this — call upon a variety of representations (Boonen et al., 2013; Kintsch & Greeno, 1985). Mathematical problem-solving further calls upon a variety of cognitive abilities including updating, inhibition, and shifting with each of these cognitive actions requiring specific conceptual, procedural, and factual knowledge and skills (Baddeley, 2000; Bull & Sherif, 2001; Lester, 2013). Gaining the necessary problem-solving experience is thus crucial for children's mathematics learning and development (Lester, 2013).

Beliefs and emotions

Children's mathematical development does not rely on cognitive factors alone but also on math-related and general learning beliefs and emotions (Chinn, 2012; Lebens et al., 2011; Giofrè et al., 2017). Math self-concept, math self-efficacy, and math anxiety have all been shown to relate to mathematical development (Ashcraft & Moore, 2009; Beilock & Maloney, 2015; Lee, 2009; Prast et al., 2018; Usher & Pajares, 2008).

Self-concept and self-efficacy both concern self-perceived competence but are distinguishable. Self-concept encompasses beliefs about one's competence and thus self-esteem (Bong & Clark, 1999).

Within the context of the present research, self-concept refers to the child's perceived mathematics competence and thus the extent to which their judgements of their own mathematics achievement match the standards they set for themselves (Arens et al., 2020; Wolff et al., 2018). Self-efficacy as conceptualized by Bandura (1997) is the child's belief in their capacity to successfully perform (in this dissertation mathematical tasks). Children with high self-efficacy beliefs are more likely than others to think of difficult tasks as challenges; have a strong commitment to their learning goals; and be willing to try out new strategies. Children with low self-efficacy beliefs do not think that they can overcome obstacles and handle threats, which leads them to avoid difficult tasks (Bandura, 1993; Op't Eynde et al., 2006). In previous research, clear associations have been found between children's math self-efficacy and mathematics achievement — especially their mathematical problem-solving (Pajares & Kranzler, 1995; Op't Eynde et al., 2006).

An emotional factor that has been found to negatively influence children's mathematical development is so-called math anxiety or a negative emotional reaction to mathematics (Suárez-Pellicioni et al., 2016). In several studies, for example, avoidance of math-related situations and suppression of cognitive processing by the experienced anxiety have been documented and thus found to contribute to a vicious negative spiral for mathematics achievement (Ashcraft, 2002; Maloney & Beilock, 2012).

Conversely in previous research, positive associations have been found between prior math self-concept and later math self-efficacy (Arens et al., 2020; Pajares & Miller, 1994). That is, children appear to base their math-specific judgements and thus self-efficacy on their previously formed and somewhat more general math self-concept. Good mathematics achievement is positively related to math self-concept and math self-efficacy and negatively to math anxiety (Marsh et al., 2005; Weidinger et al., 2018).

Teacher predictors of mathematical development

The teaching of mathematics involves longer-term learning processes. Teachers contribute to children's mathematical development with the use of effective mathematical classroom practices, including: whole class discussion, use of a range of representations and tools, making informed decisions about what to do to meet children's learning needs, and highlighting connections across different mathematical topics (Anthony & Walshaw, 2009; Hiebert, & Grouws, 2007; Kyriakides et al., 2013). Especially when it comes to adapting their teaching to the different needs of the children in their classrooms, teachers must be able to monitor child progress, understand a child's learning needs, and have the knowledge and skills needed to adapt their lessons. Such attunement requires advanced professional teaching skills and mathematical knowledge for teaching (Deunk et al., 2018; Hill et al., 2008; Prast et al., 2015).

The professional competencies of teachers can be divided into cognitive factors (e.g., mathematical knowledge for teaching) and professional beliefs (e.g., positive self-efficacy for the teaching of mathematics, motivation) (Blömeke et al., 2015; Döhrmann et al., 2012; Kaiser et al., 2017). In a number of studies, three key components of the teaching of mathematics have been shown to be associated with children's mathematics achievement: actual teaching behavior during mathematics lessons (Hiebert & Grouws, 2007; Stronge et al., 2011), teacher's mathematical knowledge for teaching (Campbell et al., 2014; Hill et al., 2005), and teacher's perceptions of their own self-efficacy for the teaching of mathematics (Perera & John, 2020; Tella, 2008).

Mathematics teaching behavior

With regard to the associations between mathematics teaching behavior and children's mathematical development, different aspects have been examined. In some studies, the manner of classroom management, attention to math concepts/misconceptions, use of interactive and activating teaching methods, and supply of individualized support have all been shown to contribute to children's mathematics achievement (Muijs & Reynolds, 2002, 2011; Stronge et al., 2011). Blazar (2015) found inquiry-oriented instruction but not classroom management or emotional support to relate to mathematics achievement. Review results showed domain-specific learning activities, time for learning, and differentiation/ adaptive instruction to all positively correlate with children's achievement (Seidel & Shavelson, 2007). The associations between teaching behavior and children's mathematics achievement are nevertheless not clear-cut across studies for the reason that the studies do not address the same aspects of teaching behavior and often emply different measures of teaching behavior and mathematics achievement (Seidel & Schavelson, 2007).

In a large-scale observational study, Van de Grift (2007) found the following variables related to the quality of teaching: a safe and stimulating learning climate; clear instruction; adaptive teaching; use of modeling, explanation, scaffolding (i.e., type of teaching strategies); and efficient classroom management. Follow-up research by Van der Lans et al. (2018) found that relevant teaching behaviors could be ranked according to level of complexity and thus from the simple provision of a safe learning climate and efficient classroom management to differentiation of learning needs and adaptation of lessons on the basis of identified needs. These complexity levels provide insight of how effective teaching develops and can also support teachers with feedback about how they can approve their effectiveness (Van der Lans et al., 2018).

Mathematical knowledge for teaching

The mathematical knowledge for teaching includes the specific mathematical knowledge and skills of teachers that are needed to effectively teach mathematics. Based on the mathematical knowledge for teaching framework of Ball et al. (2008), subject matter knowledge and pedagogical content knowledge can be distinguished. Subject matter knowledge includes common content knowledge (i.e., mathematical knowledge that is not unique to teaching and thus also useful in other professions), horizon content knowledge (e.g., seeing connections between early and later mathematics), and specialized content knowledge that is thus specific to the teaching of mathematics (e.g., understanding children's solution methods, accurately use of representations). Pedagogical content knowledge includes knowledge of content and children, knowledge of content and teaching, and knowledge of content and curriculum (Ball et al., 2008). Inconsistent results have been found for the associations of mathematical knowledge for teaching with children's mathematics achievement. Some studies show significant influences of mathematical knowledge for teaching on children's achievement (Campbell et al., 2014; Hill et al., 2005) while others do not (Muijs & Reynolds, 2002; Shechtman et al., 2010). Most studies investigated mathematical knowledge for teaching in relation to the quality of teachers' teaching in this subject matter and instructional quality in particular (Baumert et al., 2010; Hill et al., 2008). In addition, it can be concluded that not only mathematical knowledge for teaching but also teacher's math-related beliefs and attitudes clearly play a role in their mathematics teaching practices (Wilkins, 2008).

Teacher self-efficacy in relation to the teaching of mathematics

Teacher self-efficacy within the domain of mathematics teaching refers to the teacher's own perceptions of their capacity to promote children's mathematics learning, mathematics achievement, and mathematics engagement (Bandura, 1993, 1997; Tschannen-Moran & Woolfolk Hoy, 2001). Based on the process-oriented model of teacher self-efficacy as put forth by Woolfolk Hoy et al. (2009), higher levels of math achievement can be expected in classrooms where the teacher believes in their capacity to perform the tasks and conduct the activities needed to realize math learning goals. Recently, Perara and John (2020) found teachers' self-efficacy with regard to mathematics teaching to positively correlate with average class levels of achievement and teacher-student interaction quality. Nevertheless, relatively few studies have found significant associations between teacher self-efficacy and children's mathematics achievement (Ashton & Webb, 1986; Tella, 2008). And in a review study, Klassen et al. (2011) pointed out that the connections between teacher self-efficacy and children's mathematics achievement are not as strong as presumed.

To summarize, research has shown inconsistent results for the associations between — on the one hand — actual mathematics teaching behavior, teachers' mathematical knowledge for teaching, and teachers' mathematics teaching self-efficacy and — on the other hand — children's mathematical development. Greater clarity is thus needed about the specific roles of these aspects of mathematics teaching in children's mathematical development.

The role of dynamic math interviews

Individual differences in children's mathematics learning are clearly noticeable but nevertheless call for teachers who can meet a variety of math learning needs and thus constitute a major challenge for many elementary school teachers (Charalambous, 2015). Understanding children's math learning needs is a prerequisite for adapting one's teaching to the needs of children (Deunk et al., 2018; Hoth et al., 2016). To date, mostly standardized, norm-referenced, and product-focused testing has been used to identify and gain insight into math learning needs (Bodi, 2017). It is increasingly being recognized, however, that more formative assessment is called for to provide more process-focused, supplemental information on children's math learning needs (Ginsburg, 2009; Veldhuis et al., 2013). And one such form of formative assessment is the so-called dynamic math interview.

Dynamic math interviews

A dynamic math interview is a semi-structured dialogue between the teacher and children with a process-oriented character to be used in a variety of mathematics domains to identify and understand specific learning needs. The teacher gathers and analyzes information about the child's understanding of a specific learning goal to then provide supplemental instruction or some other form of support to help the child meet the learning goal (Black & William, 2009; Ginsburg, 1997, 2009). In such an individual interview, teachers can assess achievement levels, underlying knowledge, skills, learning potential, beliefs, and emotions related to mathematics (Allsopp et al., 2008; Ginsburg, 1997, 2009; Pellegrino et al., 2001; Van Luit, 2019). In interaction with the child, the teacher actively involves the child to attain responses and thereby see things from the child's point of view to identify how they can best meet the child's math learning needs (Lee & Johnston-Wilder, 2013). The interview is support-oriented and solution-oriented. This support/solution orientation is reflected in questions aimed at actively stimulating the child to think about math learning strengths, future goals, and the type of support needed to obtain these goals (Allsopp et al., 2008; Bannink, 2010; Ginsburg, 2009; Ketterlin-Geller & Yovanoff, 2009). The dynamic math interview supplements standardized norm-based testing (Allsopp et al., 2008; Franke et al., 2001; Wright et al., 2006).

The information obtained by the teacher in a dynamic math interview can also be deployed in daily mathematics instruction to — for example — design or adapt interventions within the child's so-called zone of proximal development (i.e., what a child can perform with support, but cannot yet perform on its own) to support the child's mathematics learning and problem-solving processes, and to promote child's self-confidence for mathematics learning (Bakker et al., 2015; Deunk et al., 2018; Lee & Johnston-Wilder, 2013). Examples of relevant interventions are: providing additional instruction, offering challenging tasks, using more concrete materials, and linking a new math concept to prior math concepts or experiences. The use of dynamic math interviewing can thus bridge the gap between children's math learning needs and a teacher's mathematics teaching. Although scripted protocols for dynamic math interviewing could be of assistance to the teacher to conduct such interviews, these are rarely developed (Caffrey et al., 2008).

Professional development of teachers

Dynamic math interviewing requires specific teacher competencies concerned with mathematics but also communication. Teacher must ask a variety of questions with a specific purpose in mind; create a safe and stimulating interview climate; explore and expand the limits of the child's mathematical knowledge; gain insight into the child's mathematical thinking; and stimulate the child to respond in a much detail as possible and thereby gain insight into the child's capacities and perspective (Campbell et al., 2014; Empson & Jacobs, 2008; Ginsburg, 1997, 2009; Lee & Johnston-Wilder, 2013; Mercer, 2008).

A professional development program for the introduction and use of dynamic math interviewing should be designed in keeping with what is known about effective teacher development (Heck et al., 2019; National Research Council, 2001). Such a program should entail collective participation and collaboration, active learning, a focus on content, coherence, and a sufficient investment of time and effort (Desimone, 2009; Van Driel et al., 2012).

Aims, research questions, and design of the present research

The aim of the present research was to unravel the specific roles of various child and teacher factors in children's mathematical development and, as part of doing this, the capacity of teachers to use dynamic math interviews to identify the specific math learning needs of elementary school children. The following main research questions were as follows.

1. How can children's mathematical development, specifically arithmetic fluency and mathematical problem-solving, be predicted by child and teacher factors?
2. To what extent does the use of dynamic math interviews facilitate the identification of the math learning needs of children, promote teachers' mathematics teaching and promote children's mathematics learning?

To address the first research question, the prediction of children's mathematical development – namely, arithmetic fluency and mathematical problem-solving[1] – by various child factors (entrance-level mathematics achievement, math self-concept, math self-efficacy, and math anxiety after control for non-verbal reasoning) and by various teacher factors (actual mathematics teaching behavior, mathematical knowledge for teaching, and mathematics teaching self-efficacy) was examined longitudinally. Just how a number of aspects of children's executive functioning (visuospatial and verbal updating, inhibition, and shifting) predict children's mathematics achievement and mathematical development was also then examined. The aim of these analyses was to uncover the specific contributions of relevant child and teacher factors to children's mathematical development.

To address the second research question, the utility of conducting dynamic math interviews to identify children's math learning needs and improve the teaching of mathematics was investigated quasi-experimentally. The intervention consisted of participation of teachers

[1] It should be noted that mathematical problem-solving is understood here as solving non-routine mathematical problems that thus challenge the child to come up with their own solution strategy (or strategies) (Polya, 1957; Doorman et al., 2007) In the present research, the data on the children's mathematical problem-solving was collected using problems calling for the use of mathematical notation, text, and/or pictures — as done in standard Dutch math textbooks.

in a professional development program to develop their dynamic math interviewing competencies, followed by a period of practice using dynamic math interviewing. To investigate the effectiveness of using dynamic math interviews, the reliability, validity, and further benefits of using this form of assessment to identify children's math learning needs were assessed.

An overview of the components of the research project is presented in Figure 1.

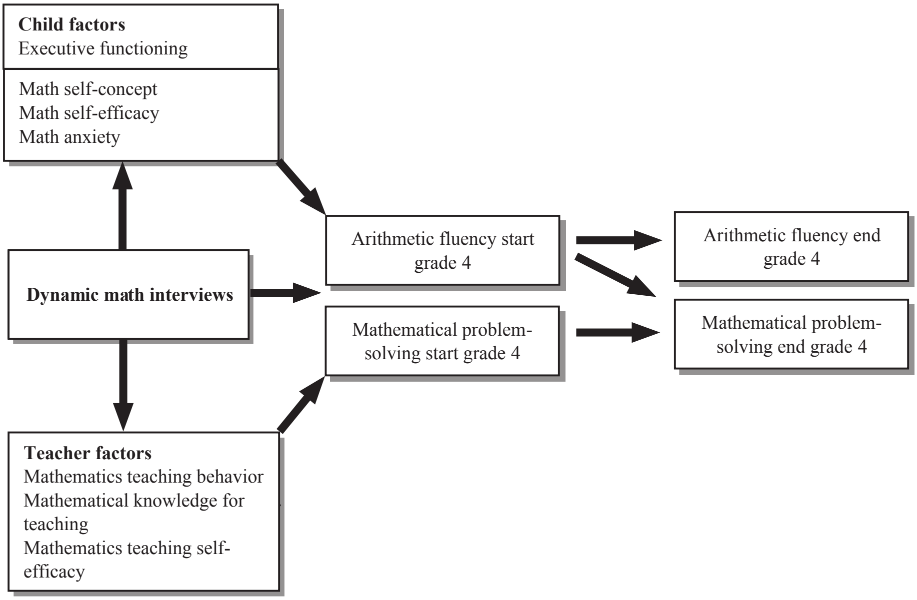

Figure 1 An Overview of the Components of the Research Project

For purposes of the present research, a professional development program was developed following the design features recommended for such programs (Borko et al., 2011; Desimone, 2009; Heck et al., 2019; Tripp & Rich, 2012; Van Driel et al., 2012). The program included an explanation of the dynamic math interview tool and the mathematical knowledge for teaching needed for dynamic math interviewing.

A support tool for the conduct of dynamic math interviews was also constructed. An analytic framework was next developed to examine those aspects of the dynamic math interviews considered critical for the effective identification of the math learning needs of elementary school children (see Appendix C).

1

Research design

The design of the present research is outlined in Figure 2. The data on child factors and teacher factors was collected at the start (T1) and end (T2) of two consecutive school years. Year 1 constituted the control condition as no intervention was conducted during that year. Year 2 constituted the experimental condition; a dynamic math interview professional development program was conducted during this year and followed by a period of practice. The same teachers participated in years 1 and 2, but the groups of children participating in the years differed.

<table>
<tr><th colspan="8">School year 1: control group</th></tr>
<tr><td>Aug-Sep</td><td>Oct</td><td colspan="3">Nov-mid Feb</td><td>Feb</td><td>March-mid June</td><td>June</td></tr>
<tr><td>Measurement 1, year 1</td><td colspan="6">Mathematics taught as usual</td><td>Measure-ment 2, year 1</td></tr>
<tr><th colspan="8">School year 2: experimental group</th></tr>
<tr><td>Measurement 1, year 2</td><td>Individual feedback on a conducted dynamic math interview</td><td>Pre test</td><td>Profes-sional develop-ment program</td><td>Post test</td><td>Individual feedback on a conducted dynamic math interview</td><td>Practice period</td><td>Measure-ment 2, year 2</td></tr>
</table>

Figure 2 Research Design

Outline of the present dissertation

In Chapter 2, the results of a longitudinal study of the roles of both child and teacher factors in children's mathematical development are reported. Not only the roles of various cognitive aspects of mathematical development but also the math-related beliefs and emotions of children were examined. And the roles of teachers' mathematics teaching behavior, and teachers perceived mathematical knowledge for teaching and their math teaching self-efficacy were examined. Data was collected from 610 fourth grade children and 31 fourth-grade teachers. In multi-level analyses, the extent to which various child and teacher factors considered separately but also jointly correlated with children's mathematical development was examined after control for non-verbal reasoning ability. For each of the analytic models, arithmetic fluency and mathematical problem-solving were distinguished.

In Chapter 3, the results are reported for a second longitudinal study examining the roles of arithmetic fluency and executive functioning (visuospatial and verbal updating, inhibition, and shifting) in children's mathematical problem-solving achievement and development, after control for non-verbal reasoning ability as this is a critical factor underlying mathematical problem-solving ability. Data were collected from a sample of 458 children randomly selected from the population of 1062 children participating in the two years of the research project. The sample was evenly distributed with respect to low-, average-, and high mathematical achieving. In multiple hierarchical regression analyses, the roles of arithmetic fluency and executive functioning in mathematics achievement at the end of grade 4 were examined. Mediation analyses were used to investigate the relationships between executive functioning and mathematical problem-solving development with the children's arithmetic fluency at the start of grade 4 as the mediator and their mathematics achievement at the start grade 4 as a covariate.

In Chapters 4 and 5, the results are reported of quasi-experimental studies with dynamic math interviews. In Chapter 4, the outcomes are reported for a professional development program aimed at enhancing the quality of the conduct of dynamic math interviews and identifying the benefits of using dynamic math interviews to pinpoint children's math learning needs. A total of 23 teachers involved in both years of the research project participated in this specific study. Data on the effects of the professional development program on the quality of dynamic math interviews was collected using pretest-posttest measures and compared using paired samples t-tests. In repeated measures ANOVA analyses, followed by post hoc tests, the effects of participation in the professional development program on teacher factors were examined.

In Chapter 5, the adequacy of teachers' conduct of dynamic math interviews and the possible benefits of using dynamic math interviews with children showing low mathematics achievement are reported on. Date was collected during the second year of the present research project (i.e., the intervention year). Participants were 19 teachers who had children showing low mathematics achievement in their classes. The teachers conducted a dynamic math interview. The interviews were video recorded, and qualitative analyses were conducted on the videos to determine the capacity of the teachers to adequately identify the children's specific math learning needs. To examine to the promotion of the children's mathematics learning using dynamic math interviews, a Wilcoxon signed-rank test was computed.

Chapter 6 provides a summary of the main findings from the four studies constituting the present dissertation and a general discussion of the findings, some critical reflection, and suggestions for future research. The practical and empirical implications of the present findings are also described in this final chapter.

Appendices describing and/or illustrating the developed instruments are also included. Appendix A contains the Scale for Mathematics Teaching Strategies supplemented to The International Comparative Analysis of Learning and Teaching (ICALT + S), Appendix B includes the Teachers' Sense of Mathematical Knowledge for Teaching Questionnaire (TSMKTQ). Appendix C contains the Analytical Framework to facilitate the qualitative analysis of the dynamic math interviews and in Appendix D examples of parts of the dynamic math interviews are presented.

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Chapter 2

Impact of child and teacher factors on mathematical development

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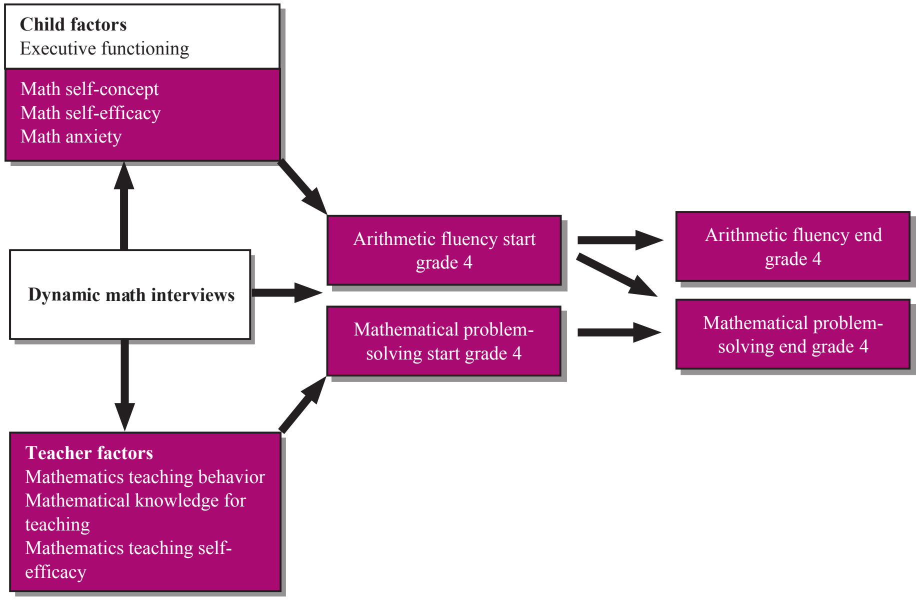
Abstract

We examined to what extent children's development of arithmetic fluency and mathematical problem-solving was influenced by their math self-concept, math self-efficacy, and math anxiety but also teacher factors, specifically: actual mathematics teaching behavior, self-efficacy, and mathematical knowledge for teaching. Participants were 610 children and 31 teachers of grade 4. Multi-level analyses showed children's math self-concept to be a positive predictor of arithmetic fluency and mathematics teaching behavior to be a negative predictor. The development of mathematical problem-solving was predicted: positively by mathematical knowledge for teaching; negatively by actual mathematics teaching behavior and teachers' self-efficacy; and not at all by the child factors of math self-concept, math self-efficacy, or math anxiety. Promoting the self-confidence of young children is essential for their mathematical development. More research into the relationship between mathematics teaching behavior and children's mathematical development is needed.



Introduction

The main goal of mathematical education today is to develop the knowledge and skills needed for later professional and personal lives (OECD, 2010; Tout & Gal, 2015). Two essential subdomains are arithmetic fluency (i.e., the ability to add, subtract, multiply, and divide fast and accurately) and mathematical problem-solving (i.e., solving problems using mathematical notation, text, and/or pictures) (National Research Council, 2001; Powell et al., 2013). Mathematics is known to be hard for some children due to such factors as low mathematical self-esteem and no appropriate mathematical education (Mazzocco, 2007).

To understand the development of children's mathematical skill, research has paid more attention to cognitive, information-processing, and neuropsychological factors and less attention to child self-perceptions and beliefs about mathematical skill. However, children's math self-concept (Bong & Clark, 1999; Timmerman et al., 2017), math self-efficacy (Bandura, 1997; Joët et al., 2011; Pajares & Miller, 1994), and math anxiety (Ashcraft & Moore, 2009; Ramirez et al., 2016) have been shown to significantly correlate with mathematics achievement. In general, better mathematics skill positively correlates with math self-concept and math self-efficacy while poorer mathematical skill negatively correlates with math anxiety. Similarly, children's mathematical development has been shown to be significantly associated with the observed mathematics teaching behavior of teachers (Muijs & Reynolds, 2000, 2002; Stronge et al., 2011), mathematical knowledge for teaching (Baumert et al., 2010; Hill et al., 2005), and teachers' mathematics teaching self-efficacy (Klassen et al., 2009; Tella, 2008).

Research has yet to consider the roles of *both* child and teacher factors together for understanding children's mathematical development. In addition, arithmetic fluency and mathematical problem-solving are not distinguished clearly in most research despite the involvement of different underlying skills. In the current study, we therefore investigated the influences of two sets of factors on the development of the arithmetic fluency and mathematical problem-solving abilities. We examined, in particular: 1) the math self-concept, math self-efficacy, and math anxiety of fourth grade children and 2) the actual mathematics teaching behavior, mathematical knowledge for teaching, and mathematics teaching self-efficacy of their teachers.

Mathematical development

During early elementary school, children are expected to develop an understanding of numbers, counting, and simple arithmetic (Geary, 2003). With increasing arithmetic speed and accuracy, a solid foundation is assumed to be laid for the development of more advanced mathematical problem-solving abilities (Gersten et al., 2005). Geary (2004) has provided a theoretical framework in which mathematical development is assumed to relate to the combined functioning of the visuospatial and language systems, the central executive functioning of the brain, conceptual development, and procedural knowledge (e.g., knowledge of rules and algorithms). Knowledge of basic arithmetic combinations is stored in long-term memory and easily retrieved for the solution of mathematical problems using short-term memory information (Baddeley, 2000). The development of arithmetic fluency and mathematical problem-solving can thus be seen as distinct aspects of children's mathematical development (Fuchs et al., 2008).

Arithmetic fluency is the ability to add, subtract, multiply, and divide with basic number combinations accurately and quickly. The development of arithmetic fluency starts with the onset of formal mathematical education. As part of early elementary education (children aged 6-8 years), considerable attention is paid to the promotion of arithmetic knowledge and fluency. The speed and accuracy of children's performance on arithmetic fact problems increases between the first and seventh grades (Ostad, 2000) with attention and processing speed identified as key factors (Fuchs et al., 2008). And the later mathematical development of children who have difficulties retrieving basic arithmetic facts from long-term memory has been shown to be hampered (Duncan et al., 2007; Geary, 2004; Geary & Hoard, 2005).

Mathematical problem-solving can be defined as the ability to apply mathematical knowledge and skills to solve actual or imagined "real life" imaginable problems using mathematical notation, text, and/or pictures. Mathematical problem-solving is taught in mainly the upper grades of elementary school. From about fourth grade (children aged 8-10 years), the focus of mathematical education shifts to advanced mathematics (e.g., fractions, proportions) and the abstractness and complexity of mathematical tasks increases. Mathematical problem-solving requires children to be able to read the problem, distinguish relevant from irrelevant information, identify key words, derive underlying numerical relationships, select and apply required operations and algorithms, and manipulate numbers procedurally (Fuchs et al., 2008; Goldin, 1998; Kintsch & Greeno, 1985). The brain's central executive system of working memory plays an important role in the integration of information for the solution of mathematical problems and has thus been found to be an important predictor of developing mathematical problem-solving ability (Swanson & Beebe-Frankenberger, 2004).

Several longitudinal studies have shown strong associations between early and later mathematics achievement (Byrnes & Wasik, 2009; Duncan et al., 2007; Watts et al., 2014). And the developments of both arithmetic fluency and mathematical problem-solving have been shown to be highly stable with early mathematical skill critical for the development of later mathematical skill (Fuchs et al., 2006; Watts et al., 2014).

There is nevertheless evidence that *additional* child and teacher factors are crucial for the development of mathematical skill.

Roles of children's math self-concept, math self-efficacy, and math anxiety

As already mentioned, children's mathematical development depends on several factors with cognitive factors receiving the most attention in previous research. Mathematical development has also been shown to relate to children's mathematical self-beliefs (Bandura, 1997; Pajares & Miller, 1994). In the first years of elementary school, children have positive and even at times unrealistic perceptions of their abilities. These early self-beliefs are relatively unstable (Wigfield & Eccles, 2000). By the age of seven/eight years, children have become more sensitive to performance feedback and their self-perceptions become more realistic and stable (Dweck, 2002).

Three aspects of math self-belief have been distinguished to date: math self-concept, math self-efficacy, and math anxiety. *Math self-concept* subsumes beliefs about self-worth associated with mathematical competence. In general, self-concept is less specific than self-efficacy (Bong & Clark, 1999). *Math self-efficacy* is a judgment of one's capacity to perform domain-specific tasks— for example — solve word mathematical problems or fact problems and succeed (Bandura, 1997). A child may have a generally positive, math self-concept but hold quite different beliefs about specific mathematics tasks (i.e., negative self-efficacy at times). *Math anxiety* is a negative emotional response to numbers and/or math-related situations (Suárez-Pellicioni et al., 2016).

Positive correlations have generally been found between math self-concept and mathematics achievement (McWilliams et al., 2013; Möller et al., 2009). Viljaranta et al. (2014) did not, however, find math self-concept to *predict* subsequent mathematics achievement. Timmerman et al. (2017) found positive correlations between math self-concept and both arithmetic fluency and mathematical problem-solving in adolescents. Previous experiences with mathematical problem-solving can obviously contribute to math self-concept (Elbaum & Vaughn, 2001), while math self-concept can conversely influence mathematical performance (Marsh et al., 2005). By grade 4, reciprocal associations have indeed been found with children's self-concept significantly influencing their mathematics achievement and vice versa (Weidinger et al., 2018).

Children's *experience* with mathematical tasks in the past has been shown to be most influential for math self-efficacy (Usher & Pajares, 2008, 2009). In addition, the receipt of efficacy-related information including positive social messages about mathematical performance and evaluative feedback from teachers but also experienced emotional states and physiological reactions have been shown to significantly influence math self-efficacy (Bandura, 1997; Joët et al., 2011). Furthermore, Pietsch et al. (2003) have shown math self-efficacy to correlate more strongly with mathematics achievement than math self-concept does. Pajares and Kranzler (1995) showed math self-efficacy, moreover, to be predictive of mathematics achievement in general and mathematical problem-solving in particular.

Lee (2009) found clear cross-cultural differences when she examined all three aspects of math self-belief in conjunction with the mathematics achievement of 276,165 children aged 15 years using PISA 2003 questionnaire data from 41 countries. The strongest associations between math self-concept and mathematics achievement were found in Western European countries. The strongest associations between math self-efficacy and mathematics achievement were found in Asian and Eastern European countries. The associations between math anxiety and mathematics achievement were stronger in Western and Eastern European countries than in Asian countries. And some of the Western European countries, including the Netherlands, showed particularly low levels of math anxiety.

Inconsistent findings have nevertheless been found for math anxiety in relation to young children's mathematics achievement (Dowker et al., 2016). Math anxiety was found to negatively correlate with mathematics achievement due to avoidance of mathematics, the suppression of cognitive processing by anxiety, and/or the roles of social factors (e.g., teachers' and parents' own math anxiety) (Ashcraft, 2002; Maloney & Beilock, 2012). Math anxiety has been shown to interfere with working memory and thereby have a strong effect on mathematics achievement (Ashcraft & Kirk, 2001). Thoughts about how badly one is doing or may do (i.e., aspects of math anxiety) can distract attention from the task at hand and overload working memory at the same time. Timmerman et al. (2017) nevertheless found no significant associations between math anxiety and arithmetic fluency. With regard to mathematical problem-solving, however, Ramirez et al. (2016) found math anxiety to indeed be a negative predictor of the adoption of advanced problem-solving strategies and a positive predictor of lower achievement for mathematical problem-solving. They also found both the math anxiety and mathematical problem-solving strategies to be strongest for the children with the greatest working memory capacity in the same study. In sum, mathematical difficulties and experiences of failure during the early school years can elicit and increase math anxiety. As a consequence, children may avoid further learning in the domain of mathematics, acquire increasingly more negative experiences with mathematics, and become more anxious with regard to mathematics. A vicious cycle thus emerges.

Most of the aforementioned research was cross-sectional, which precludes the drawing of conclusions about causal relations between — on the one hand — math self-concept, math self-efficacy, and math anxiety and — on the other hand — mathematics achievement. Most of the relevant studies concerned only high school students, moreover. And most of the studies considered only one aspect of self-belief (i.e., math self-concept or math self-efficacy or math anxiety) in connection with mathematics achievement.

Role of teacher factors

As might be expected, teacher characteristics and competencies can influence children's mathematics achievement. In research, three specific teacher factors have been examined in relation to children's mathematics achievement: the actual behavior of the teacher during mathematics lessons (e.g., Stronge et al., 2011), teacher's mathematical knowledge for teaching (e.g., Campbell et al., 2014), and teacher's self-efficacy with respect to the teaching of mathematics (e.g., Klassen et al., 2009).

When Van de Grift (2007) observed 854 mathematics lessons of teachers of nine year old children, the following teacher variables were found to play a critical role in children's mathematics achievement: a safe and stimulating learning climate, clear instruction, adapted teaching, type of teaching and learning strategies (e.g., model, explain, scaffold), and efficient classroom management. When Stronge et al. (2011) compared outcomes of observed lessons with data on teacher effectiveness, they found classroom management but also the relationships with children to correlate most strongly with mathematics achievement. In contrast, Blazar (2015) found no associations of classroom climate and classroom management with mathematics achievement. He found instead that inquiry-orientated instruction positively predicted children's achievement. Reynolds and Muijs (1999) found that both whole-class interactive and collaborative group-based teaching positively influenced achievement for a range of mathematical skills. In another study, Muijs and Reynolds (2002) found effective teacher behavior (e.g., interactive mathematics teaching, direct instruction), positive self-efficacy beliefs, and good subject knowledge to significantly correlate with children's mathematics achievement. Noteworthy, they found constructivist mathematics teaching to negatively correlate with mathematical development. In other research, Wenglinsky (2000) concluded that the use of hands-on learning activities to illustrate mathematical concepts and stimulate higher-order thinking skills can promote mathematics achievement. Hiebert and Grouws (2007) concluded, based on their review, that teacher behavior is effective if teachers are explicit about learning goals, make their teaching behavior dependent on the mathematical learning goal, and foster engagement particularly on the part of children who are struggling with mathematics. Teaching behavior that facilitates the development of understanding of mathematical concepts and makes the connections between ideas, facts, and procedures sufficiently explicit was found to be important for children's mathematical development (e.g., interactive instruction, think-stimulating activities, comparison of solution strategies, critical thinking). A meta-analysis focusing on teaching factors related to children's outcomes (Kyriakides et al., 2013) showed children's achievement to not be associated with a single teaching approach (e.g., direct vs. constructivist instruction); making well-considered choices and adoption of elements of different approaches were found to be crucial instead.

In observational research specifically concerned with the influences of teacher behavior on arithmetic fluency, Kling and Bay-Williams (2014) found giving children opportunities to notice relationships, adopt strategies, and practice with these strategies to promote arithmetic fluency. Muijs and Reynolds (2000) found active, whole-class teaching that clearly involves children to be associated with better achievement in arithmetic fluency. Teacher behaviors considered together, moreover, explained the basic mathematics achievement of children while individual teacher behaviors did not (e.g., organization, time spent on interactive teaching).

Regarding mathematical problem-solving, instruction focused on strategies for solving different types of problems and direct teaching of higher-level cognitive strategies were shown to improve achievement (Verschaffel et al., 1999; Wenglingsky, 2000).

Mathematical knowledge for teaching concerns knowledge of required mathematical concepts, possible misconceptions on the part of children, effective instructional strategies, and various representations. Mathematical knowledge for teaching is subject-specific and content knowledge forms a necessary prerequisite for the connection of pedagogy with context (Depaepe et al., 2013). Hill et al. (2005) found teachers' mathematical knowledge for teaching to positively predict gains in children's mathematics achievement during the first and third grades. Similarly, Campbell et al., (2014) found teachers' mathematical knowledge for teaching to directly and positively relate to children's mathematics achievement in grades 4 through 8. In a study by Muijs and Reynolds (2002), in which they collected data indirectly through a self-perception questionnaire, mathematical content knowledge correlated strongly with teachers' self-efficacy beliefs and only to a lesser extent with children's mathematical development.

Teaching self-efficacy refers to teachers' perceptions of their capacity to promote children's learning, achievement, and engagement (Bandura, 1993, 1997; Tschannen-Moran & Woolfolk Hoy, 2001). In a review by Klassen et al., (2011), ambiguous results were found for associations between teachers' self-efficacy and general children's achievement. In other research, however, Tella (2008) found teachers' self-efficacy to contribute significantly to children's mathematics achievement. Ashton and Webb (1986) also found a positive correlation between teachers' self-efficacy and children's mathematics achievement.

The present study

Despite the widespread availability of research addressing the impact of teacher-related factors on children's achievement, relatively little is known about the influence of specific teacher factors on children's mathematics performance. Research that takes a) the actual mathematics teaching behavior of teachers, b) their mathematical knowledge, and c) their mathematics teaching self-efficacy into account is quite scarce. Basic arithmetic fluency is rarely distinguished from later mathematical problem-solving, moreover. And consideration of the aforementioned factors *together* in a single study has yet to be done. In the present study, we thus examined the influences of specific teacher factors *together* with children's math self-concept, math self-efficacy, and math anxiety on children's mathematical development over time. A longitudinal design was adopted to allow us to monitor children's mathematical development from the start to the end of the fourth grade.

The general research question was: How do a) children's math self-concept, math self-efficacy, and math anxiety, b) teacher factors, and c) combinations of these child and teacher factors predict the development of children's arithmetic fluency and mathematical problem-solving during the course of the fourth grade?

We expected, even after control for the children's entrance level mathematical abilities, both the child and teacher factors to make unique contributions to the development of both arithmetic fluency and mathematical problem-solving.

Method

Participants and study context

Participants were recruited via social media (Twitter) and letters to both elementary school principals and fourth grade teachers (contact information gathered via public websites for schools). Two-thirds of those approached responded to the open invitation, which included information on the aims of the study, what was expected of the participants, and what the participants could expect of the researchers. In the end, 31 teachers agreed to participate and the study was conducted during the 2016-17 school year in the Netherlands.

The teachers worked with 610 children at 27 elementary schools located in different parts of the Netherlands. The sizes of the schools varied: 6% had fewer than 100 children (small); 66% had between 100-400 children (medium); and 28% had more than 400 children (large).

The composition of the classes varied: 66% homogeneous (all fourth grade); 34% heterogeneous (combination of two grades in one class). The mean age of the teachers was 38;1 (years; months) (range of 24 to 60 years) with 16% male and 84% female. The majority of the teachers had a bachelor's degree in education (66%); 28% had additional graduate training; and 6% had a Master's degree in education. The teachers had an average of 11.9 years of experience (*SD* = 8.7) (range of 2 to 39 years).

Of the 610 children, 53% was male and 47% female. The age of the fourth graders ranged from 8;2 to 10;10 with a mean of 9;2 (*SD* = 0.31). The wide spread in age was due to either having skipped a year or having stayed behind a year. The home language for 88.5% of the children was Dutch.

The children's non-verbal reasoning was tested using the Raven's Standard Progressive Matrices (SPM). It was checked that none of the children scored two or more standard deviations below the mean (Raven, 2000; Raven et al., 1998). None of the children did. The mean nonverbal reasoning score for the children was 36.64 (*SD* = 7.43), skewness -0.86, kurtosis 1.51.

Measurement instruments

Mathematics achievement

Children's mathematics achievement was measured using two instruments: a test of arithmetic fluency (addition, subtraction, multiplication, and division) and a test of advanced mathematical problem-solving (fact and word problems).

Arithmetic fluency. The Speeded Arithmetic Test (TTA; De Vos, 2010) is a standardized paper-and-pencil test frequently used in Dutch and Flemish education to measure speeded arithmetic skill (arithmetic fluency). The test consists of four categories of 50 fact problems: addition (tasks with a difficulty level varying from 6 + 0 to 29 + 28), subtraction (difficulty level varying from 4 – 2 to 84 – 38), multiplication (difficulty level varying from 4 x 1 to 7 x 9), and division (difficulty level varying from 6 : 2 to 72 : 9). Children are given two minutes per category of problems. Each correct answer yields one point, for 50 possible points per category and a total possible score of 200. The total score was used in the analyses. The reliability and validity of such testing has been found to be good (α = .88; De Vos, 2010), also in this study (α = .79).

Mathematical problem-solving. Children's mathematics achievement was measured using the criterion-based mathematics tests (Cito; Janssen et al., 2005), which are standardized Dutch national test commonly administered at the middle and end of each school year to monitor children's progress. The test consists of a mixture of mathematical problems in several domains presented in varied ways: only using mathematical notation or combinations of text, mathematical tasks related pictures, and mathematical notation as used in regular curricula (e.g., *There are 24 boxes in a warehouse. Each box contains 8 cans of soup. How many cans of soup are there?*). The following domains are covered: 1) numbers, number relations, and operations (addition, subtraction, multiplication, and division), 2) proportions and fractions, and 3) measurement and geometry. The reliability coefficients for the tests have been found to range from .91 to .97 (Janssen et al., 2010). In this study the internal consistency was found to be good (α = .82 start grade 4 and α = .84 end grade 4).

Child factors: emotions and beliefs

The *math self-concept*, *math self-efficacy*, and *math anxiety* of the children were measured using the Mathematics Motivation Questionnaire for Children (MMQC; Prast, et al., 2012). The questionnaire consists of five scales: math self-efficacy (6 items), math self-concept (6 items), mathematical task value (7 items), math lack of challenge (6 items), and math anxiety (5 items). Items are rated along a four-point scale: 1 = NO! (strongly disagree), 2 = no (disagree), 3 = yes (agree), 4 = YES! (strongly agree). A sample item from the math self-concept scale is "Are you good in mathematics?". A sample item from the math self-efficacy scale is "When the teacher explains the first mathematical problem, are you capable of solving the next math problem by yourself?". A sample item from the math anxiety scale is "Are you afraid to make mistakes during the mathematics lesson?". These three scales were used in the present study and their internal consistency was found to be good (self-concept α = .91; self-efficacy α = .81; math anxiety α = .79).

Teacher factors

Actual teaching behavior in mathematics lessons. The actual teaching behavior of the teachers in their mathematics lessons was measured using the International Comparative Analysis of Learning and Teaching (ICALT), an observation instrument (Van de Grift, 2007). The ICALT, consisting of seven scales, covers many aspects of teaching behavior and is not math-specific. For purposes of the present study, the instrument was therefore supplemented with an eighth scale specifically addressing the teaching of mathematics (see Appendix A). The ICALT itself involves 32 items addressing six aspects of teaching behavior ranging from lower order teaching behavior to higher order teaching behavior (Van der Lans et al., 2015, 2018): a) safe and stimulating learning climate, b) efficient classroom management, c) quality of instruction, d) activation of children, e) teaching of learning strategies, and f) differentiation/ adaptation of lesson content to meet children's math learning needs. The seventh scale addresses children's involvement. The eighth scale addressed math-specific teaching strategies using the following 8 items: a) informal manipulation, b) representations of real objects and situations, c) abstract mental representations (models and diagrams), d) abstract concepts/mental operations, e) connecting these four levels and using these appropriate to the goal of the lesson, pay attention to f) planning, g) solving processes, and h) metacognitive skills. All of the scales used in the present study were found to have reliable Cronbach's alphas. The internal consistency of the ICALT has been found in the past to be good ($\alpha = .82$). The internal consistency of the ICALT with the supplemental scales (ICALT+S) used in the present study was similarly found to be good ($\alpha = .85$).

Mathematical knowledge for teaching. Teachers' mathematical knowledge for teaching was self-assessed using a questionnaire specifically developed for the present study: the Teachers' Sense of Mathematical Knowledge for Teaching Questionnaire (TSMKTQ; Kaskens et al., 2016; see Appendix B). Composed of three parts and 38 questions, the following are assessed: a) mathematical skill in the domains of numbers, number relations and operations, proportions and fractions, measurement and geometry (*Subject Matter Knowledge*); b) ability to follow and analyze children's thinking including recognition of errors and responding to these (*Pedagogical Content Knowledge*); and c) selection and use of models and representations for different domains of mathematics, use of real-world contexts, and knowledge of the metric system (*Specialized Content Knowledge*). Teachers responded to items along a four-point scale ranging from 1 (= *to a very small extent*) to 4 (= *to a very large extent*). The internal consistency of the TSMKTQ was found to be good (α = .93).

Teachers' mathematics teaching self-efficacy. The Dutch online version (Goei & Schipper, 2016) of the long form of the Teachers' Sense of Self Efficacy Scale (TSES; Tschannen-Moran & Woolfolk-Hoy, 2001) was used to measure teachers' self-efficacy with respect to the teaching of mathematics. The questionnaire contains 24 items equally divided across three subscales: a) efficacy for children's engagement (e.g., *How much can you do to help children think critically?*), b) efficacy for instructional strategies (e.g., *How well can you respond to difficult questions from your children?*), and c) efficacy for classroom management (e.g., *How much can you do to get children to follow classroom rules?*). The teachers responded along a nine-point scale ranging from 1 (= *not at all*) to 9 (= *a great deal*). Reliability was found to be good in the present study: the Cronbach's alphas for the three subscales were 0.74, 0.81, and 0.82, respectively.

Procedure

After recruitment of participants, an information meeting was held in two different regions of the Netherlands. During the meeting, the teachers were given written information about the study and a factsheet about the methods of data collection to be used. The teachers consented via e-mail for subsequent observation and video-recording of a regular mathematics lesson taught by them on the topic of fractions or ratios.

The parents of children were provided written information about the study by the teacher. Their written consent for participation of their child in the study was obtained prior to data collection. The sample was treated in accordance with institutional guidelines as well as with APA ethical standards.

Data collection

As part of a larger longitudinal research project, data of children and teachers were obtained on two measurement occasions: at the start of the school year (in September-October) (= T1) and at the end of the school year (in May-June) (= T2).

Teachers. The TSMKTQ and TSES were sent to the 31 participating teachers using the web-based questionnaire services of Formdesk (TSMKTQ and TSES). An email was sent with a direct link to the Formdesk questionnaires and the teachers were asked to complete the two questionnaires. This was done at the beginning and the end of the school year with two reminders sent on each occasion. Response rate was 100%; all collected data from the 31 teachers was thus included in subsequent analyses.

For purposes of observation (and video recording), the teachers were asked to teach as normal as possible in order to provide representative data. It was agreed that the topic of the lesson would be in the domain of fractions or proportions. In accordance with the procedure of Van de Grift et al. (2014), the ICALT+S observations were conducted by two trained observers. The training consisted of an explanation of the observation instrument, group discussions, and the rating of three video-recorded sample lessons. For each sample lesson, observers scored the 40 items from the ICALT+S along a 4-point Likert scale ranging from 1 (= *predominantly weak*) to 4 (= *predominantly strong*). Observers who met the consensus norm of .70 or higher were judged to be sufficiently qualified. All of the observed mathematics lessons were also video recorded. The inter-rater reliability for live scoring was good (0.86). The first author conducted 65% of the observations; a fellow observer conducted the remaining observations.

On the same day as the ICALT observation of the teacher, data was collected from the children.

Children. The MMQC, TTA, and RAVEN were conducted using paper and pencil in the class, with one examiner giving instructions. The teacher remained in the classroom. Children were positioned in a test setup so that they were not able to copy from one another. The examiner remained in the classroom at all times to answer any questions. The procedure lasted approximately 65 minutes (excluding breaks, which were arranged for the children and taken periodically).

The Cito mathematics achievement data were obtained from the participating teachers, with parental consent. The test scores at the end of grade 4 were used as the outcome measure of mathematics achievement; the test scores at the start of grade 4 were used as a baseline measure. It must be noted that the baseline measure was actually included as part of standardized testing at the end of grade 3, but for clarity and consistency we are using this as the level at the start of grade 4.

The participating teachers were debriefed after measurement and thus informed of results. Due to illness or other reasons for school absence, relocation to a new school during the school year, or incomplete test responding, the number of data points for the children per test varied from 525 to 610.

Data analyses

The data and descriptive statistics for all of the measures were first screened for potential errors and outliers. Three separate multilevel models were then operationalized to examine: a) the extent to which child-related factors influence their mathematical development (model 1); b) the extent to which teacher-related factors influence mathematical development (model 2); and c) the extent to which child- and teacher-related factors considered together influence mathematical development (model 3). The models were structured incrementally. And in each of the three models, Arithmetic Fluency (AF) and mathematical Problem-Solving (PS) were distinguished as individual measures of mathematics achievement.

In a two-level hierarchical structure, arithmetic fluency (AF) (N = 525) (T2) and mathematical problem-solving (PS) (N = 576) (T2) were nested within teacher/class (N= 31). Given the nested structure of the data (i.e., children within classes) and the sample size of 31 teachers/classes, we therefore decided to first investigate whether multilevel modelling was actually needed. The intra-class correlation (ICC) and the design effect (Deff) were computed with the mixed model procedure of SPSS 25.0. The sample sizes at the classroom level were relatively small, which meant that restricted maximum likelihood (RML) estimation was employed (Hox, 2010). For completeness, maximum likelihood

(ML) estimation and restricted maximum likelihood (REML) estimation were compared, but the ICC and Deff were equal.

The multilevel models were built according to the procedures of Heck et al. (2014) and Peugh (2010). All of the analyses started from the unconditional models in which the mean levels of the dependent variables were estimated while taking into account the variances at the levels of child and teacher/classroom. The unconditional “null” models were used to test the multilevel structure of the data. Subsequent models were then built including all predictors (“full” model). Nonsignificant predictors were next removed from the models to create the final “restricted” models. The fit indices for the final models were compared to those for the unconditional models to determine model improvement. A deviance statistic (-2 log likelihood) was calculated to decide if model fit improved. The deviance statistic had a large sample chi-square distribution, with degrees of freedom equal to the between-model difference in the number of parameters estimated. The significance of the improvement in model fit was tested using a χ^2 difference test. For mathematics achievement AF, the ICC was 0.10 and Deff 2.51. For mathematics achievement PS, ICC was 0.255 and Deff 5.48. Because the ICCs > 0 and the Deffs > 2 (Peugh, 2010), multilevel linear models were tested in all of the subsequent analyses. Continuous predictor variables were grand mean centered.

Results

Descriptive statistics

The means, standard deviations, and ranges for the different measures are presented in Table 1. All variables were normally distributed, with skewness and kurtosis within the normal ranges (Tabachnick & Fidell, 2013). Before turning to the research question, we also established that the mathematics achievement of the children indeed increased during the school year. Paired samples *t*-tests showed higher scores at the end of the school year than at the beginning for the two measures of mathematics achievement: (arithmetic fluency, $t(519) = 19.92$, $p < .001$, $d = 0.57$; problem-solving $t(552) = 20.18$, $p < .001$, $d = 0.77$).

Table 1 Measures of Child and Teacher Factors

	Child				
	N	*M* (*SD*)	Range	Skewness	Kurtosis
Arithmetic fluency T1	610	105.22 (35.72)	(9-185)	0.19	-0.65
Arithmetic fluency T2	525	125.81 (34.72)	(34-196)	-0.11	-0.62
Math. problem-solving T1	586	217.43 (26.08)	(131-321)	-0.14	0.52
Math. problem-solving T2	576	237.77 (26.35)	(84-319)	-0.57	1.91
Math self-concept T1	605	20.40 (5.37)	(7-30)	-0.44	-0.60
Math self-efficacy T1	605	17.79 (3.45)	(7-28)	-0.35	0.22
Math anxiety T1	605	11.41 (4.25)	(6-24)	0.85	0.16
	Teacher				
Actual teaching behavior	31	2.86 (0.25)	(2.39-3.38)	-0.01	-0.81
Math. knowledge for teaching	31	3.15 (0.30)	(2.47-3.87)	-0.17	0.13
Math teaching self-efficacy	31	7.08 (0.44)	(6.13-7.96)	-0.31	-0.71

Pearson's correlation coefficients were next computed between the various child and teacher factors (Table 2). All of the child measures correlated significantly with the child mathematics achievement measures. In addition: actual mathematics teaching behavior correlated significantly with mathematical problem-solving at the end of the year (T2); mathematical knowledge for teaching correlated significantly with both arithmetic fluency at the start of the year (T1) and mathematical problem-solving at the start of the year (T1); and the mathematics teachers' self-efficacy correlated significantly with their actual mathematics teaching behavior, on the one hand, and their mathematical knowledge for teaching, on the other hand.

Children's math self-concept, math self-efficacy, and math anxiety as predictors of mathematical development

The first part of our research question concerns the extent to which the children's mathematical development during fourth grade was predicted by their math self-concept, math self-efficacy, and math anxiety when measured at the start of the school year. To answer this question, multi-level analyses were computed separately for the children's Arithmetic Fluency (AF) and mathematical Problem-Solving abilities (PS).

Table 2 Correlations Between Child and Teacher Factors

Measure	1	2	3	4	5	6	7	8	9	10
1. Arithmetic Fluency T1	-									
2. Arithmetic Fluency T2	833**	-								
3. Math. Problem-Solving T1	.529**	529**	-							
4. Math. Problem-Solving T2	467**	463**	773**	-						
5. Math self-concept	534**	512**	.552**	.473**	-					
6. Math self-efficacy	.465**	428**	.440**	.394**	910**	-				
7. Math anxiety	-.300**	-.303**	-.349**	-.265**	-.598**	-.563**	-			
8. Math. teaching behavior	.055	-.038	-.047	-.152**	.024	.033	.026	-		
9. Math. knowledge for teaching	-.083*	-.064	-.092*	.021	-.034	-.033	.019	-.317**	-	
10. Math. teaching self-efficacy	-.039	.024	-.004	.003	.004	.009	.002	-.388**	.301**	-

Note: Math. = mathematical/mathematics; **Correlation significant at 0.01 level (2-tailed); *Correlation significant at 0.05 level (2-tailed)

Table 3 Children's Math Self-concept, Math Self-efficacy, and Math Anxiety as Predictors of Mathematical Development

	Model 1 AF Unconditional	Model 1 PS Unconditional	Model 2 AF Level 1 Full model	Model 2 PS Level 1 Full model	Model 4 AF Level 2 (class)	Model 4 PS Level 2 (class)
			Regression coefficients (fixed effects)			
Intercept	125.81** (1.52)	237.77*** (1.10)	44.61* (26.51)	76.26*** (13.70)	43.22*** (3.23)	76.96*** (6.63)
Prior achievement			0.77*** (0.28)	0.74*** (0.03)	0.78*** (0.03)	0.74*** (0.03)
Math self-concept			1.64** (0.53)	0.28 (0.46)	1.71*** (0.52)	0.43 (0.39)
Math self-efficacy			-0.88 (0.59)	0.32 (0.51)	-1.01 (0.54)	0.33 (0.44)
Math anxiety			0.15 (0.25)	0.17 (0.21)	0.30 (0.25)	0.17 (0.17)
			Variance components (random effects)			
Intercept variance class					56.09 (0.00)	59.18 (40.17)
Prior achievement Variance					9.25 (0.00)	0.00 (0.00)
Math self-concept Variance					0.98 (0.54)	0.00 (0.00)
Math self-efficacy Variance					0.00 (0.00)	0.24 (0.29)
Math anxiety Variance					0.25 (0.36)	0.00 (0.00)
Variance part. ICC	0.10	0.26	0.11	0.22	0.14	0.31
-2 Log Likelihood	5210.83	5400.14	4458.58	4588.85	4395.38	4417.80

Note: * $p < .05$, ** $p < .01$, *** $p < .001$.
AF = Arithmetic Fluency; PS = mathematical Problem-Solving; Math. = mathematical/mathematics

For Arithmetic Fluency (AF), the unconditional model with AF (T2) as dependent variable showed the level 1 mathematics achievement scores of the children to vary significantly. To create the full model, all of the predictors were added into the unconditional model as fixed effects: that is, prior AF achievement (i.e., the initial measurement of AF, T1), math self-concept, math self-efficacy, and math anxiety. The full model showed a deviance statistic (-2 log likelihood) of 4458.58, indicating that the fit was significantly better than that provided by the unconditional model (i.e., the model not including these predictors) (β, = 752.25, $p < .001$). Prior achievement ($M = 0.77$, $SD = 0.28$, $p < .001$) and math self-concept ($M = 1.64$, $SD = 0.53$, $p < .01$) were significant predictors of AF (T2). Math self-efficacy ($M = -0.88$, $SD = 0.59$, $p = 0.14$) and math anxiety ($M = 0.15$, $SD = 0.25$, $p = 0.54$) were not. This level-1 full model explained 11% of the total variance in the children's AF, T2 (ICC = 0.11).

We next computed the restricted model by removing all nonsignificant predictors from the model (in this case: math self-efficacy and math anxiety). The level-1 restricted model did not provide a better fit for the data relative to the level-1 full model (β 0 = 44.32, $SD = 3.08$, $p < .001$; prior AF achievement $M = 0.77$, $SD = 0.03$, $p < .001$; math self-concept $M = 0.87$, $SD = 0.24$, $p < .001$; ICC = 0.11); the outcomes for the restricted model are therefore not presented in Table 3. In order to control for nesting within teacher/class, we finally computed the random effects for level 2 (class). Measures of children's development AF were thus corrected for the possible influences of teacher/class. Prior achievement ($M = 0.78$, $SD = 0.03$, $p < .001$) and math self-concept ($M = 1.71$, $SD = 0.52$, $p < .001$) continued to be significant predictors. This model explained 14% of the total variance in the children's AF, T2 (ICC = 0.14).

The same analyses were conducted for the children's mathematical Problem-Solving (PS). The coefficients and ICCs for the different models are presented in Table 3. The unconditional model showed the level-1 mathematics achievement (PS) scores of the children to vary significantly. When all of the predictor measures were added to the unconditional model as fixed effects to create a full model, a deviance statistic (-2 log likelihood) of 4588.85 was found, showing the fit of the full model to be significantly better than the fit of the unconditional model (β, = 811.29, $p < .001$). Prior PS achievement (i.e., the initial measurement of PS, T1) ($M = 0.74$, $SD = 0.03$, $p < .001$) significantly predicted PS achievement, T2. The children's math self-concept ($M = 0.28$, $SD = 0.46$, $p = 0.55$), math self-efficacy ($M = 0.32$, $SD = 0.51$, $p = 0.54$), and math anxiety ($M = 0.17$, $SD = 0.21$, $p = 0.42$) were not found to be significant predictors. This level-1 full model explained 22% of the total variance in the children's PS, T2 (ICC = 0.22).

When the restricted model was created by removing all nonsignificant predictors (i.e., math self-concept, math self-efficacy, and math anxiety), a better fit was not obtained (β 0 = 69.29, $SD = 5.93$, $p < .001$; prior PS achievement $M = 0.77$, $SD = 0.03$, $p < .001$; ICC = 0.20); the outcomes for the restricted model are therefore not presented in Table 3. In order to control for nesting within teacher/class, we finally computed the random effects for level 2 (class). Measures of children's PS development were thus corrected for the possible influences of teacher/class. Prior PS achievement was again the only significant predictor ($M = 0.74$, $SD = 0.03$, p < .001). This restricted model explained 31% of the total variance in the children's PS, T2 (ICC = 0.31).

Teacher competencies as predictors of children's mathematical development

To examine how mathematical development in grade 4 is predicted by teacher competencies, we conducted multi-level analyses that examined actual mathematics teaching behavior, mathematical knowledge for teaching, and mathematics teaching self-efficacy when measured at the start of the school year in relation to children's arithmetic fluency (AF, T1 and T2) and mathematical problem-solving (PS, T1 and T2).

For AF, we first computed the unconditional model (see Table 4 for the coefficients and ICCs). The unconditional model showed the level-1 AF scores of the children to vary significantly. The full model was next created by adding children's prior AF achievement and all of the teacher measures to the unconditional model as fixed effects.

Table 4 Teacher Competencies as Predictors of Mathematical Development

	Model 1 AF Unconditional	Model 1 PS Unconditional	Model 2 AF Level 1 Full model	Model 2 PS Level 1 Full model	Model 4 AF Level 2 (class)	Model 4 PS Level 2 (class)
Regression coefficients (fixed effects)						
Intercept	125.81*** (1.52)	237.77*** (1.10)	38.41*** (13.70)	68.52 *** (5.83)	38.08*** (2.79)	66.36*** (5.33)
Prior achievement			0.83*** (0.02)	0.78*** (0.03)	0.83*** (0.02)	0.79*** (0.02)
Math. teaching behavior			-11.34 ** (3.66)	-10.65*** (3.02)	-14.98* (7.28)	-13.85 (9.12)
Math. teaching knowledge			-3.64 (3.11)	8.85*** (2.55)	-6.43 (6.45)	0.57 (6.02)
Math. teaching self-efficacy			2.56 (2.10)	-5.29 ** (1.70)	0.22 (4.24)	-3.66 (4.12)
Variance components (random effects)						
Intercept variance class					36.85 (24.29)	0.00 (0.00)
Prior achievement					0.00 (0.00)	0.00 (0.00)
Math. teaching behavior Variance					0.00 (0.00)	916.79 (545.56)
Math. teaching knowledge Variance					0.00 (0.00)	0.00 (0.00)
Math. teaching self-efficacy Variance					0.00 (0.00)	0.00 (0.00)
Variance part. ICC	0.10	0.26	0.10	0.21	0.11	0.27
-2 Log Likelihood	5210.83	5400.14	4517.05	4632.60	4473.74	4479.27

Note: * $p < .05$, ** $p < .01$, *** $p < .001$.
AF = Arithmetic Fluency; PS = mathematical Problem-Solving; Math. = mathematical/mathematics

2

The full model showed a deviance statistic (-2 log likelihood) of 4517.05, indicating that the fit of the full model is significantly better than that of the null model (β, = 693.78, $p < .001$). Children's prior AF achievement was, as might be expected, a significant predictor of their AF development ($M = 0.83$, $SD = 0.02$, $p < .001$). Mathematics teaching behavior was significantly but negatively related to AF development ($M = -11.34$, $SD = 3.66$, $p < .01$). Neither mathematical knowledge for teaching related significantly to the development of AF ($M = -3.64$, $SD = 3.11$, $p = 0.24$) nor mathematics teaching self-efficacy ($M = 2.56$, $SD = 2.10$, $p = 0.23$).

When the restricted model was computed by removing all nonsignificant predictors of AF (in this case: mathematical knowledge for teaching and mathematics teaching self-efficacy), a better fit was not obtained ($\beta_0 = 38.30$, $SD = 7.73$, $p < .001$; prior AF achievement $M = 0.83$, $SD = 0.02$, $p < .001$; actual mathematics teaching behavior $M = -12.07$, $SD = 3.22$, $p < .001$; ICC = 0.10); the outcomes for this restricted model are therefore not included in Table 4. The level-1 full model still provides the best fit with the inclusion of children's prior AF achievement and measures of actual teaching behavior, mathematical knowledge for teaching, and mathematics teaching self-efficacy together explaining 10% of the total variance in the children's AF (ICC = 0.10). In order to control for nesting within teacher/class, we finally computed the random effects for level 2 (class). The χ^2 change for this model including class variance with mathematics teaching behavior, mathematical knowledge for teaching, and mathematics teaching self-efficacy was significant ($\chi^2 = 43.31$, $p < .001$). This model explained 11% of the total variance in the children's development AF (T1 and T2) (ICC = 0.11).

The same analyses were conducted to examine the influences of teacher competencies on the development of children's mathematical PS (see Table 4). The unconditional model showed the level-1 PS scores of the children to vary significantly. To create the full model, children's prior PS achievement and all three teacher measures were added to the unconditional model as fixed effects. The full model showed a deviance statistic (-2 log likelihood) of 4632.60, indicating a significantly better fit for the full model (β, = 767.54, $p < .001$). As could be expected, the children's prior PS achievement significantly predicted their later PS achievement (*M* = 0.78, *SD* = 0.03, $p < .001$). In addition, all three teacher measures showed significant connections to children's mathematical development (PS): actual mathematics teaching behavior was negatively related (*M* = -10.65, *SD* = 3.02, $p < .001$); mathematical knowledge for teaching was positively related (*M* = 8.85, *SD* = 2.55, $p < .001$); and mathematics teaching self-efficacy was negatively related to children's later mathematical PS (*M* = -5.29, *SD* = 1.70, $p < 0.01$). This level-1 full model with the children's prior PS achievement included together with all of the teacher measures explained 21% of the total variance in the children's mathematical development (i.e., mathematical PS, T1 and T2) (ICC = 0.21). The computation of a restricted model was not necessary.

Finally, we computed the random effects for level 2 (class) in order to control for nesting within classes for PS. This model showed a deviance statistic (-2 log likelihood) of 4479.27, which indicates added value. The χ^2 change proved significant for this model taking variance due to teacher/class into account (χ^2 =153.33, $p < .001$). The nested model including mathematics teaching behavior, mathematical knowledge for teaching, and mathematics teaching self-efficacy explains 27% of the total variance in the children's development PS (ICC = 0.27).

Child and teacher factors as predictors of children's mathematical development

We computed multilevel models to examine the influences of all of the child and teacher factors considered together on the children's fourth-grade mathematical development. For arithmetic fluency (AF), we started with an unconditional model and found the level-1 AF scores of the children to vary significantly (Table 5). When we calculated the full prediction model, a deviance statistic (-2 log likelihood) of 4429.68 was found, showing the full model to fit significantly better than the unconditional model (β, = 644.26, $p < .001$). This level-1 full model — containing all child and teacher factors — explained 11% of the total variance in the development of AF (T1, T2) (ICC = 0.11). We computed a restricted model by removing all nonsignificant predictors from the full model; only prior AF achievement, children's math self-concept, and mathematics teaching behavior remained in the restricted model. The level-1 restricted model did not provide a better fit (β_0 = 43.75, *SD* = 17.68, *p* < .001; prior AF achievement *M* = 0.78, *SD* = 0.03, *p* < .001; children's math self-concept *M* = 0.86, *SD* = 0.23, *p* < .001; teachers' actual teaching behavior *M* = -12.39, *SD* = 3.19, *p* < .001 (ICC = 0.11); the outcomes are therefore not included in Table 5. In order to control for nesting within teacher/class, we computed the random effects for level 2 (class). This model, in which children's AF development is corrected for the possible influences of teacher/class, provided the best fit (ICC = 0.13). Significant predictors were now prior AF achievement (*M* = 0.63, *SD* = 0.17, *p* < .001) and the children's math self-concept (*M* = 1.63, *SD* = 0.77, *p* < .05). Level-2 analyses showed an added class value of 2% relative to that for the full level-1 model.

Mathematical development assessed in terms of mathematical problem-solving (PS) was analyzed next. In the initial unconditional model , the level-1 PS scores of the children were found to vary significantly (Table 5). For the full PS model, with all of the child and teacher factors included as fixed effects, a deviance statistic (-2 log likelihood) of 4545.89 was found, indicating that the full model provided a significantly better fit than the unconditional model (β, = 539.27, *p* < .001). The full model — containing all child and teacher factors — explained 23% of the total variance in the children's PS (T1, T2) (ICC= 0.23).

We next computed a restricted model by removing all nonsignificant child and teacher factors from the full model; this meant removal math self-concept, math self-efficacy, and math anxiety for the children. This level-1 restricted model — now including all teacher factors in addition to the prior PS achievement of the children — did not provide a better fit than the full model (β_0 = 68.52, *SD* = 5.83, *p* < .001; prior PS achievement *M* = 0.78, *SD* = 0.03, *p* < .001; teachers' actual teaching behavior *M* = -10.65, *SD* = 3.02, *p* < .001; mathematical knowledge for teaching *M* = 8.85, *SD* = 2.55, *p* < .001; self-efficacy *M* = - 5.28, *SD* = 1.70, *p* < .01 (ICC = 0.21). The results for the restricted model are therefore not included in Table 5. In order to control for nesting within teacher/class, we finally computed the random effects for level 2 (class). This nested model with children's PS mathematical development corrected for the

Table 5 Children's Math Self-concept, Math Self-efficacy, and Math Anxiety together with Teacher Competencies as Predictors of Children's Mathematical Development

	Model 1 AF Unconditional	Model 1 PS Unconditional	Model 2 AF Level 1 Full model	Model 2 PS Level 1 Full model	Model 4 AF Level 2 (class)	Model 4 PS Level 2 (class)
	Regression coefficients (fixed effects)					
Intercept	125.81*** (1.52)	237.77*** (1.10)	44.09 *** (7.61)	75.18*** (7.04)	42.49*** (3.38)	81.22*** (7.08)
Prior achievement			0.77*** (0.28)	0.75*** (0.03)		
Math self-concept			1.66** (0.52)	0.20 (0.45)		
Math self-efficacy			-0.88 (0.59)	0.48 (0.50)		
Math anxiety			0.21 (0.24)	0.21 (0.20)		
Math. teaching behavior			-11.83*** (3.64)	-11.11*** (3.03)		
Math. knowledge for teaching			-3.48 (3.08)	8.85** (2.55)		
Math. teaching self-efficacy			2.36 (2.09)	-5.42** (1.70)		
Variance components (random effects)						
Intercept variance class					0.00 (0.00)	63.08 (378.19)
Prior achievement Variance					0.63*** (0.17)	0.57*** (0.15)
Math self-concept Variance					1.63* (0.77)	0.16 (0.43)
Math self-efficacy Variance					0.00 (0.00)	0.62 (0.64)
Math anxiety Variance					0.06 (0.33)	0.00 (0.00)
Math. teaching behavior Variance					829.78 (1154.25)	10296.21 (7334.59)
Math. knowledge for teaching Variance					0.00 (0.00)	0.00 (0.00)
Math. teaching self-efficacy Variance					363.09 (445.30)	0.00 (0.00)
Model summary						
Variance part. ICC	0.09	0.26	0.11	0.23	0.13	0.33
-2 Log Likelihood	5073.94	5085.16	4429.68	4545.89	4527.21	4577.08

Note: * $p < .05$, ** $p < .01$, *** $p < .001$.
AF = Arithmetic Fluency; PS = mathematical Problem-Solving; Math. = mathematical/mathematics.

2

possible influences of teacher/class provided a better fit than just the level-1 full model (ICC = 0.33). The prior PS achievement of the children was now the only significant predictor (M = 0.57, SD = 0.15, p < .001). The level-2 analyses showed an added class value of 10% relative to that for the full level-1 model.

Discussion

In this study, we investigated longitudinally the prediction of the development of arithmetic fluency and mathematical problem-solving during the fourth grade for some 600 children. This was done on the basis of their math self-concept, math self-efficacy, and math anxiety but also the teacher competencies of actual mathematics teaching behavior, mathematical knowledge for teaching, and mathematics teaching self-efficacy.

For the development of arithmetic fluency, both the children's arithmetic fluency at the start of fourth grade and their math self-concept were found to be significant positive predictors; mathematics teaching behavior was found to be a significant negative predictor.

With regard to the development of mathematical problem-solving, both the children's mathematical problem-solving at the start of fourth grade and the teachers' mathematical knowledge were significant positive predictors; mathematics teaching behavior and mathematics teaching self-efficacy were significant negative predictors.

Child and teacher factors as predictors of mathematical development

Child factors

We expected children's math self-concept, math self-efficacy, and math anxiety to predict the development of both children's arithmetic fluency and mathematical problem-solving ability in grade 4. This expectation was tentative as previous studies typically involved older-aged children (e.g., McWilliams et al., 2013; Pietsch et al., 2003;

Timmerman et al., 2017) and produced inconsistent results. Out of the child factors, only math self-concept was found to be a significant predictor of arithmetic fluency in the present study, which aligns with the previous outcomes of Timmerman et al. (2017). Children's math self-concept is generally more past-oriented and stable than children's math self-efficacy, which — by definition — concerns the future (Möller et al., 2009). The influence of math self-concept on the development of arithmetic fluency, in particular, can therefore probably be explained by the fourth grade children having greater experience with arithmetic than with mathematical problem-solving (Dweck, 2002; Marsh et al., 2005; Weidinger et al., 2018). In the lower elementary school grades, considerable attention is paid to basic arithmetic skills and understandably less attention to mathematical problem-solving.

We did not find children's math self-efficacy to significantly predict any of their mathematical development, which is not consistent with the findings of older research (Pajares & Kranzler, 1995; Pietsch et al., 2003; Usher & Pajares, 2008, 2009). It is possible that math self-efficacy only predicts later development and thus development beyond fourth grade when children are better able to assess and align their expectations with regard to what they think that they can accomplish in specific mathematical tasks (Pajares & Miller, 1994). In other words, elementary school children's self-efficacy within the domain of mathematical is still malleable and can therefore be enhanced during their school careers — a possibility to be considered along with just how and when to do this in future research.

Math anxiety was also not found to be a significant predictor of any aspect of the children's mathematical development. A possible explanation for this finding is that math anxiety has been found to generally increase during childhood (Dowker et al., 2016; Ma, 1999) and therefore probably not found to influence mathematical development at the age of fourth grade children. An alternative explanation is that children in these schools experienced encouraging environments and thus developed positive mathematics attitudes as a result (Beilock & Maloney, 2015).

The finding that children's mathematical problem-solving was not influenced in the present study by the children's math self-concept, math self-efficacy, or math anxiety is in contrast to the findings of previous research (Pajares & Kranzler, 1995; Ramirez et al., 2016). This led us to explore the results for low achievers in the present study, but the results of multilevel analyses showed no significant differences between this group of children and the total group of children.

Teacher factors

Just as for the child factors, we also found results contrary to what was expected for the influence of teacher factors on the children's fourth-grade mathematical development. Although previous research has shown positive associations between mathematics teaching behavior and children's mathematics achievement (e.g., Blazar, 2015; Reynolds & Muijs, 1999; Stronge et al., 2011; Van de Grift, 2007), we found only negative associations between mathematics teaching behavior and the children's development (i.e., arithmetic fluency and mathematical problem-solving). This is in line with research that also found negative associations (Muijs & Reynolds, 2002).

This surprising negative influence of mathematics teaching behavior on children's mathematical development might be due, at least in part, to the nature of elementary mathematics education in the Netherlands (Hickendorff et al., 2017). Elementary mathematics education in the Netherlands is characterized by a mixture of learning in contexts intended to encourage mathematical understanding and the practice of basic skills. Textbooks give teachers an important guideline for the identification and attainment of specific mathematical goals. This teaching has been shown to start out well in the Netherlands (Hickendorff et al., 2017), but also call for a dynamic classroom context. Different mathematics strengths, needs, and developmental pathways are encountered during elementary mathematics teaching and call for additional teacher competencies, such as the ability to adapting mathematics lessons and to conduct micro-interventions (Corno, 2008). Some teachers may simply not be able to respond effectively to the math learning needs of the children they are teaching. In older research, for example, Stipek et al. (2001) found teachers to believe that they should fully control instruction and focus primarily on the acquisition of the skills, rules, and procedures needed to achieve correct performance rather than being focused on spontaneous learning, diverse thinking processes and mathematical understanding of children, which requires adaptive teacher competencies.

The teaching of mathematics is known to be complicated, involve longer-term learning processes, and indeed call for teachers to adapt their teaching to the different needs of the children in their classrooms (Ball et al., 2008; Corno, 2008). Muijs and Reynolds (2002) found that teachers perceive themselves to have more content knowledge and skills for teaching in the early mathematics domains compared to later domains of the mathematical curriculum (e.g., fractions and proportions). This suggests that teachers are aware of the importance of having sufficient mathematical knowledge for teaching. With regard to the influence of the teachers' mathematical knowledge for teaching, this was indeed found to be the case: it significantly predicted the development of the children's mathematical problem-solving in the present study. This finding is in line with the assumption that specific mathematics competencies are required of teachers to teach and stimulate mathematical problem-solving (Kolovou, 2011; Walshaw & Anthony, 2008). Although mathematics teaching behavior that facilitates arithmetic fluency or mathematical problem-solving overlaps, specific accents are required. The development of arithmetic fluency requires teaching behavior that is aimed at the selection of appropriate problem-solving strategies in mathematics and practice with these strategies. This can generally be achieved using active, whole-class teaching (Kling & Bay-Williams, 2014; Muijs & Reynolds, 2000). In contrast, the development of mathematical problem-solving requires that the teacher pose think-activating questions, clearly verify solutions for children, be sensitive to the math learning needs of the children, flexible enough to meet the individual math learning needs of children, and capable of checking that math learning goals have been achieved (Hiebert & Grouws, 2007; Van der Lans et al., 2015, 2018; Verschaffel et al., 1999).

Finally and again contrary to what was expected on the basis of several previous studies (Ashton & Webb, 1986; Joët et al., 2011; Pietsch et al., 2003; Tella, 2008), the mathematics teaching self-efficacy of the teachers *negatively* related to the development of the children's mathematical problem-solving and showed no significant associations with the development of their arithmetic fluency. These results suggest that the mathematics teaching self-efficacy of teachers may depend on the subdomain of mathematics in question and whether, for example, they are being asked to stimulate arithmetic fluency or more abstract mathematical problem-solving. Teachers may not recognize the complexity of mathematical problem-solving for children and what the teaching of this requires. It is apparently difficult for teachers to identify what is necessary and apply this in more advanced mathematics teaching situations.

According to Hiebert and Grouws (2007), a number of factors can hinder the development of effective mathematics teaching behavior, such as a lack of not only subject matter knowledge but also the necessary pedagogical knowledge to teach mathematics flexibly, and the absence of a useful knowledge base for teachers to improve their mathematics teaching practices.

The Dunning-Kruger effect (Kruger & Dunning, 1999) might also be at play: less competent teachers fail to recognize their incompetency in teaching mathematical problem-solving. Self-assessment of mathematics teaching self-efficacy by particularly teachers with a lower level of mathematics teaching competence can actually lead to overestimation of their capacity to promote the development of mathematical problem-solving on the part of children.

In the models in which we combined child and teacher factors with control for the possible influences of teacher/class on mathematical development, the results resembled those for the models in which child-related factors and teacher-related factors were distinguished.

Study strengths, limitations, and directions for further research

The present study involved a large sample of more than 500 children but a relatively small sample of 31 teachers. Caution is thus warranted when generalizing the results to other teachers.

First, we measured math self-concept, math-self-efficacy, and math anxiety in the manner used by others, namely by administration of a written self-perception questionnaire (e.g., Joët et al., 2011; McWilliams et al., 2013; Pajares & Kranzler, 1995; Ramirez et al., 2016). It is nevertheless possible that some of the fourth grade children had difficulties responding to the questionnaires in writing their responses as opposed to other methods of measuring such as oral response on a questionnaire. One recent exception is a study by Viljaranta et al. (2014) in which a written self-concept scale was used in combination with the posing of a single question by an interviewer with fourth grade children and just a written questionnaire with seventh grade children. They still found children's math self-concept to not be predictive of subsequent mathematics achievement. In addition, the limited number of questions used to address math self-concept, math self-efficacy, and math anxiety limit the generalizability of the present results. In future research, alternative means of measurement and using a greater number of questions, should be considered.

Second, the use of exclusively quantitative methods to assess both the teacher and child factors may not have fully captured the underlying character of the factors. Observational rating, for example, may not capture the richness of actual behavior during the teaching of a mathematics lesson. Some examples of information that might have been missed are the exact nature of the questions posed by the teachers, the reaction of the teachers when the children adopt an approach that differs from the expected approach to solving a mathematical problem, or the use of specific mathematical terminology by the teachers. The adoption of both quantitative and qualitative measures in the future might thus be fruitful (Lund, 2012). In such a manner and as recommended by Kyriakides et al. (2013), exactly what the teacher and the children do during a mathematics lesson can be explored along with just how they interact. Another limitation to mention is that the outcome measure of mathematics teaching behavior is at the classroom level while our measures of the child factors are at the individual level.

Finally, observation of only a mathematics lesson concerned with fractions and proportions may have limited our results. The teaching of various domains of mathematics should thus be examined in the future and thereby allow us to compare the teaching of arithmetic fluency with the teaching of mathematical problem-solving. In line with the design of the present study, it is important in future research to recognize the possible specificity of the influences of various child and teacher factors depending on the particular domain of mathematics teaching and mathematics task being considered.

Implications for practice

The present results have shed light on the roles of various child and teacher factors in the mathematical development of fourth grade children. The findings have some clear implications for the practice of mathematics education.

First, prior mathematics achievement was shown to contribute to both arithmetic fluency and mathematical problem-solving, which is in line with the findings of previous studies (Fuchs et al., 2006; Watts et al., 2014). Teachers should more clearly recognize the crucial role that they play in establishing a solid mathematics base for elementary school children to build their further learning on. Teachers should be given a better understanding of exactly which aspects of their teaching are most effective for achieving given math learning goals and thereby making more informed decisions for the achievement of these learning goals (Hiebert & Grouws, 2007). A solid mathematics foundation in the lower elementary school grades or, in other words, early proficiency with numbers and numerical operations is a prerequisite for supplementing, refining, and deepening children's mathematical knowledge, skill, and understanding (Byrnes & Wasik, 2009; Duncan et al., 2007).

Second, it is important to stimulate children's learning of new math concepts, the expansion of their mathematical knowledge, and the mastery of more advanced mathematics skills on the basis of prior learning and ability (National Research Council, 2001). Unfortunately, the best means to achieve these objectives are not completely clear. In any case, the results of the present study suggest that teachers must have not only sufficient mathematical knowledge but also sufficient pedagogical knowledge and mathematics teaching self-efficacy to do this.

In addition, teachers should be encouraged as part of their professional development to attend more to the self-concepts of their children in general and their math self-concepts in particular. Once formed, negative self-perceptions can be very persistent (Swann, 2012). A clear association between children's math self-concept and arithmetic fluency was found in the present study, showing that it is crucial to provide the best opportunities for children to learn mathematics early and feel confident about their mathematics learning.

Conclusion

This study is one of the first to examine the joint influences of several child and teacher factors on children's mathematical development over the course of a school year while distinguishing basic arithmetic fluency from more abstract mathematical problem-solving.

The findings support the assumption that children's math self-concept can clearly influence their mathematical development and, in particular, the development of their arithmetic fluency in fourth grade. Children's prior mathematics achievement was consistently the best predictor of their later mathematics achievement in the various models tested by us. Establishment of a solid mathematical foundation early in elementary school is thus critical for the subsequent development of children's mathematical knowledge and skill.

As might be expected, the teachers' own mathematical knowledge played an important role in the children's mathematical development in the present study, in particular in the development of mathematical problem-solving. Actual mathematics teaching behavior during a mathematics lesson, however, was *negatively* associated with the development of both the children's arithmetic fluency and mathematical problem-solving. In addition, the teachers' mathematics teaching self-efficacy *negatively* related to the children's mathematical problem-solving. These unexpected results with regard to the influence of specific teacher factors and self-perceptions on elementary school children's mathematical development raise some intriguing questions about the classroom teaching of mathematics. How can teachers better attune their teaching to the mathematics levels and needs of the children in their classrooms? How can teachers become more conscious of their mathematics teaching behavior, enhance their mathematics teaching competence, and become more confident about their mathematics teaching in the end?

To summarize, the present study generated new knowledge for both the theory and practice of teaching elementary mathematics. The results show the importance of promoting mathematical self-confidence on the part of young children by giving them a solid mathematics foundation for later learning. Further research on the influence of specific aspects of mathematics teaching on specific aspects of children's mathematical development is necessary to expand our knowledge of how we can best promote mathematical development in both the early and later years of elementary school.

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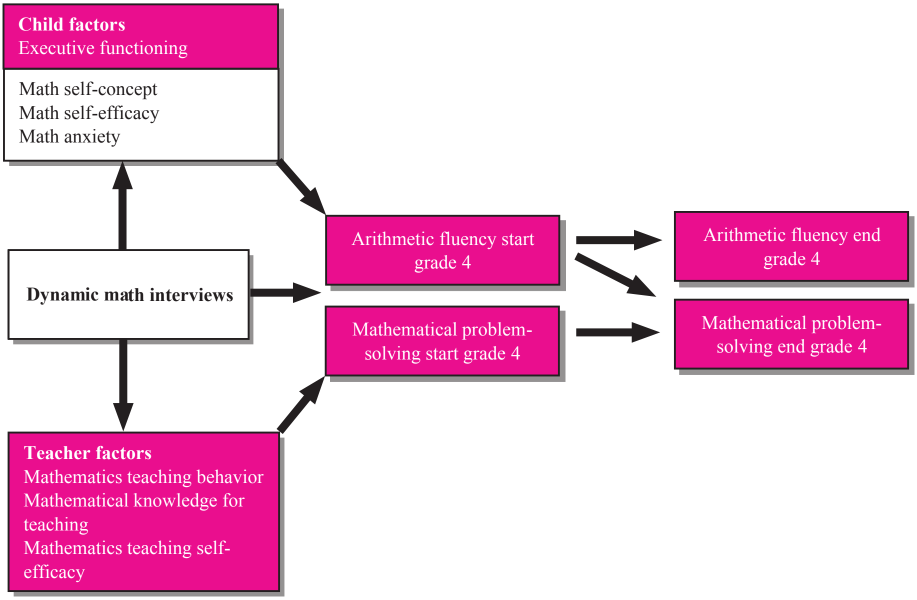
Chapter 3

Role of executive functioning in mathematical development

A manuscript, based on this chapter, has been submitted for publication.

Abstract

This study is conducted to further understand the direct and indirect contributions of executive functioning (visuospatial updating, verbal updating, inhibition, shifting) and arithmetic fluency to mathematical problem-solving in 458 fourth grade children. Arithmetic fluency along with visuospatial and verbal updating were significant predictors of mathematical problem-solving at the end of grade 4. When the development in mathematical problem-solving during the course of grade 4 was analyzed, only arithmetic fluency directly and strongly contributed to children's problem-solving at the end of grade 4. Inhibition and shifting (in combination with inhibition) were indirectly connected to the children's problem-solving at the end of grade 4 via their arithmetic fluency. Arithmetic fluency plays a critical role and continues to do this in mathematical problem-solving. Building a solid mathematical foundation during the early elementary years is therefore necessary in teaching mathematics. Furthermore, a decline in importance for visuospatial updating and verbal updating and increasing importance of inhibition and shifting (in combination with inhibition) were found with regard to children's ability to solve mathematical problems during grade 4. Teachers could consider the role of specific executive functions that might help children to solve mathematical problems and could provide appropriate support to children when teaching mathematics.



Introduction

Analytical thinking and mathematical reasoning abilities contribute to the development of problem-solving skills (Gravemeijer et al., 2017). Both arithmetic fluency (Fuchs et al., 2006, 2016; Geary, 2004; Swanson & Beebe-Frankenberger, 2004) and executive functioning (Lee et al., 2009; Viterbori et al., 2017) have been shown to be predictive for mathematical problem-solving. In some studies, mathematical problem-solving has been understood as solving non-routine mathematical problems that challenge children to come up with their own solution strategy or strategies (Doorman et al., 2007; Polya, 1957). Mathematical problem-solving has mostly been assessed using single-step or multi-step word problems "that are better simulations of the modeling problems people encounter in their personal or professional lives" (Verschaffel et al., 2020, p. 2). The scope of the present research is mathematical problem-solving defined as solving problems with mathematical notation, text, and/or pictures, which have been commonly seen in mathematics education.

However, most of the relevant research has focused on only the mathematical problem-solving of relatively young children (up to third grade or the age of about 7 years; e.g., Rasmussen & Bisanz, 2005; Swanson et al., 2008). As a result, only the solution of simple, single-step math problems has been studied (e.g., Fuchs et al., 2006; Swanson & Beebe-Frankenberger, 2004; Zheng et al., 2011). Relatively little is known about the predictive roles of arithmetic fluency and executive functioning for advanced mathematical problem-solving. However, both of these are important in light of the complexity of problem-solving tasks requiring advanced mathematical problem-solving and multi-step calculations for their solution. In addition, in grade 4 new domains of mathematics are being taught that also include certain necessary knowledge and skills (e.g., mastery of multiplication and fractions). Development of advanced mathematical reasoning and analytic thinking may not be a matter of simply mastering the required mathematical knowledge; it is possible that there is also a need for sufficient arithmetic fluency and executive cognitive functioning. Additional research on the roles of arithmetic fluency and executive functioning in the mathematical problem-solving skill of *older* elementary school children is thus needed.

Arithmetic fluency and mathematical problem-solving

During early elementary school, teachers focus on number, counting, and simple arithmetic competence (Geary, 2011). Children gradually master key arithmetic facts for quick and accurate responding (Andersson, 2008; Fuchs et al., 2006). When solving more advanced mathematical problems, children must be able to quickly retrieve these arithmetic facts from long-term memory and store this information in short-term memory (Baddeley, 2000). To be able to solve mathematical problems, it is necessary that children understand mathematical concepts (conceptual knowledge), know the procedural steps to solve a problem (procedural knowledge) and have sufficient knowledge of basic facts (factual knowledge; Geary, 2004, 2011; Geary & Hoard, 2005). Cragg et al. (2017) offered a framework presenting a refined hierarchical structure for mathematical development, based on the framework of Geary (2004). In that framework, the underlying cognitive system that supports factual knowledge, procedural knowledge, and conceptual understanding also plays a crucial role in advanced mathematical problem-solving. In light of that hierarchical structure, studies presenting both simple, single-step mathematical problems and more complex, multi-step problems have demonstrated clear associations between arithmetic fluency and mathematical problem-solving (Fuchs et al., 2006; Viterbori et al., 2017; Zheng et al., 2011). In other words, arithmetic fluency or knowing key arithmetic facts accurately and quickly (addition, subtraction, multiplication, division) has been shown to be crucial for more advanced mathematical problem-solving.

Role of executive functioning

Along with domain-specific factual knowledge, procedural skill, and conceptual understanding, domain-general cognitive skills also contribute to mathematics achievement. Many studies involving primary school-aged children have shown consensus on at least three components of executive cognitive functioning that are critical for advanced mathematical problem-solving: updating of information, inhibition of information, and shifting of attention (Bull & Lee, 2014; Miyake et al., 2000).

With regard to the updating of information, a distinction can be made between visuospatial and verbal updating (see also Baddeley, 2000). Visuospatial updating refers to the ability to monitor, manipulate, and retain information presented in a visual form or as objects in space, while verbal updating involves the ability to monitor, manipulate, and retain information presented in a verbal auditory form. Inhibition is the ability to suppress irrelevant information and/or inappropriate responses. Shifting is the capacity for flexible thinking and adeptly switching between alternative tasks or strategies (Miyake et al., 2000).

Executive functioning has been found to be linked to both arithmetic fluency and mathematical problem-solving in several ways. During the mathematical problem-solving process, information must be held in memory, manipulated, and regularly updated (Best & Miller, 2010; Bull & Lee, 2014). A representation of the required problem-solving strategy must be formed and stored in working memory. Irrelevant information or inappropriate, misleading responses must be ignored at times and alternative strategies must be considered and switched to, on occasion. Just how — and the extent to which — visuospatial and verbal updating, inhibition, and shifting (i.e., three important components of executive functioning) contribute to children's developing mathematical problem-solving is not completely clear.

Executive functioning in relation to arithmetic fluency

With practice, the arithmetic fluency of elementary school children increases, and their mathematical problem-solving becomes more efficient and sophisticated as a result (Geary, 2004). Arithmetic fluency requires not only the quick and accurate retrieval of arithmetic facts from long-term memory, but also the efficient updating of information, the suppression of incorrect responding (inhibition), and accurate shifts between operations (+, -, x, :; Bull et al., 1999; Bull & Scerif, 2001; Swanson & Beebe-Frankenberger, 2004). Consider for example, a child who has to solve 6 x 8 and needs an intermediate step. The child is able to use the strategy of splitting the problem into subproblems (5 x 8, 1 x 8). The well-known arithmetic fact that 5 x 8 = 40 has to be retrieved from memory and the child has to keep the answer in mind. Then, the child has to complete the other subproblem (1 x 8 = 8) and switch operations by adding the outcomes (40 + 8) to produce the answer to 6 x 8. During this process, the child has to inhibit responses that may have already been activated or other irrelevant stimuli (e.g., suppressing the answer 14 for the number combination of 6 and 8).

Considerable insight has been gained into the associations between executive functioning and arithmetic fluency. In particular, a number of studies have shown that visuospatial and verbal updating are significant predictors of arithmetic fluency (e.g., Cragg et al., 2017; Lee & Bull, 2016; Le Fevre et al., 2013; Van de Weijer-Bergsma et al., 2015). However, studies have shown inconsistent findings with regard to the role of visuospatial and verbal updating in relation to age/school grade. In two studies involving only verbal updating, no significant associations with arithmetic fluency were found (Balhinez & Shaul, 2019; Fuchs et al., 2006). In the study by Balhinez and Shaul (2019), moreover, verbal updating was not related to arithmetic fluency in third grade but was in the grades before. Their explanation was that young children who have to solve simple arithmetic problems possibly use different procedures that rely particularly on verbal updating. During the first years of school, arithmetic is based on the representation of a given number quantity through serial counting. Verbal updating plays an important role in arithmetic performance. When strategies become more efficient and children keep practicing, they get faster and more accurate. Arithmetic fluency mastery relies mainly on automatic retrieval and to a lesser extent on verbal updating.

In a study in which visuospatial and verbal updating were included in the analyses, Andersson (2008) found that verbal updating contributed to arithmetic fluency. Longitudinal studies have shown associations between visuospatial and verbal updating and arithmetic fluency, but the studies have not shown consistent findings. In a study by LeFevre et al. (2013), visuospatial and verbal updating jointly predicted arithmetic fluency in grades 2 through 4. Van de Weijer-Bergsma et al. (2015) showed visuospatial and verbal updating to be equally strong predictors of arithmetic fluency through grade 4 with verbal updating later prevailing in grades 5 and 6. In this same study, however, the updating of information showed no significant connections to individual differences in the development of arithmetic fluency within one school year. Finally, Lee and Bull (2016) also showed visuospatial and verbal updating to jointly and strongly predict arithmetic fluency through grade 4 but only weakly thereafter (i.e., in grades 5 through 9). Assuming that arithmetic fluency has fully developed by the end of grade 4, the authors suggest that updating also then has a less prominent role to play.

With regard to the contribution of inhibition and shifting to arithmetic fluency, previous research showed mixed findings. Several studies found relationships between inhibition and arithmetic fluency (Bull & Scerif, 2001; Cragg et al., 2017; LeFevre et al., 2013: Van der Sluis et al., 2007), but a study by Balhinez and Shaul (2019) did not. In the study by Bull and Scerif (2001), shifting was shown to contribute to arithmetic fluency, but in other studies shifting was not shown to be related to arithmetic fluency (Cragg et al., 2017; Van der Sluis et al., 2007). The mixed findings with regard to particularly the roles of inhibition and shifting in arithmetic fluency may be due to the increasingly quick and easy retrieval of stored arithmetic facts from long-term memory, making inhibition less needed and facilitating the shifting required for more complex mathematical problem-solving (Bull et al., 1999; Bull & Scerif, 2001; Cragg et al., 2017).

Executive functioning in relation to mathematical problem-solving

Mathematical problem-solving requires the following skills, among others: identification of relevant information and key words after the reading of a problem and selection and application of most suitable strategies, operations, and algorithms across multiple contexts (Boonen et al., 2013; Fuchs et al., 2008; Verschaffel et al., 2020). School textbooks typically have children solve mathematical problems involving real world contexts depicted using mathematical notation, text, and/or pictorial representations (Verschaffel et al., 2020). Visuospatial and verbal updating have indeed been found to help children integrate the information identified as relevant to thereby solve advanced mathematical problems requiring multiple steps (Cragg et al., 2017). Inhibition and shifting may also be required when learning new concepts and mastering the procedures needed for new domains of mathematics and for solving more complex mathematical problems as is the case in grade 4. To prevent irrelevant information from interfering with a new and otherwise unfamiliar problem-solving process, for example, inhibition is needed. In addition, children must be able to readily shift between various procedures for more advanced mathematical problem-solving, such as applying conceptual knowledge of fractions and factual knowledge of addition and multiplication when solving a multi-step problem (Lee et al., 2009).

The roles of visuospatial and verbal updating in mathematical problem-solving appear to be most consistent. Studies consistently report significant associations of visuospatial and verbal updating with not only simple, single-step mathematical problem-solving (Swanson, 2011; Swanson & Beebe-Frankenberger, 2004; Swanson et al., 2008; Zheng et al., 2011) but also more complex, multi-step mathematical problem-solving (Agostino et al., 2010; Cragg et al., 2017; Fuchs et al., 2016; Passolunghi & Pazzaglia, 2004). In addition, Cragg et al. (2017) found both visuospatial and verbal updating to play similar roles across different components of mathematics and different age groups. In contrast, St. Clair-Thompson and Gathercole (2006) found only visuospatial updating to be strongly related to mathematical problem-solving performance.

The few studies examining inhibition and/or shifting as executive functions in relation to children's mathematical problem-solving have shown mixed results (Jacob & Parkinson, 2015). Regarding inhibition, Lee et al. (2009) found no significant associations for multi-step problem-solving. In two other studies, in contrast, significant associations were found between inhibition and the solving of both single- and multi-step mathematical problems (Passolunghi & Pazzaglia, 2004; Swanson, 2011). Specifically, children showing better inhibition of irrelevant information showed better mathematical problem-solving. To date, the evidence regarding the role of shifting in children's mathematical problem-solving is limited and mixed. Some studies (Andersson, 2007; Cantin et al., 2016) found shifting to be a significant predictor of mathematical problem-solving, while Cragg et al. (2017) did not.

Finally, the possible associations of updating, inhibition, and shifting — considered together — with children's advanced mathematical problem-solving have only been examined in a few studies (Agostino et al., 2010; Cragg et al., 2017; Viterbori et al., 2017). The findings have again been consistent with regard to the predictive role of updating, but not about the roles of inhibition or shifting. Agostino et al. (2010) found not only visuospatial and verbal updating but also inhibition (and not shifting) to be significant predictors while Cragg et al. (2017) found only visuospatial and verbal updating (and not inhibition and shifting) to be significantly related to mathematical problem-solving. Viterbori et al. (2017) found inhibition and shifting to play a role while third graders devised a problem-solving plan and selected the required calculations but not during their actual problem-solving. When the *accuracy* of their actual mathematical problem-solving was examined, only verbal updating played a role. And similarly in a very recent study in which only updating was included, Allen and Goifré (2021) found verbal updating to play a more important role than visuospatial updating in the mathematical problem-solving of third grade children (7-8 years old).

Overall, updating is most frequently identified as a significant predictor of mathematical problem-solving performance and thus children's ability to update, hold, and manipulate information deemed to be essential. Verbal updating is judged to be of particular importance for grade 4 children. Not all studies distinguish between visuospatial and verbal updating, however. And the findings regarding inhibition and shifting in relation to child's mathematical problem-solving are less consistent than those for updating. It should be noted that when both updating and inhibition were examined in the same study, updating played a more prominent role in the children's mathematical problem-solving (Wiley & Jarosz, 2012).

Relationships between visuospatial and verbal updating, inhibition, and shifting and mathematical problem-solving performance *and* development in grade 4 are not yet clear. Most of the relevant research has included only young children and only simple as opposed to more complex mathematical problems. Very little is known about the direct and indirect contributions of executive functioning and arithmetic fluency to mathematical problem-solving in grade 4 when the degree of mathematical complexity and abstraction increases.

The present study

To date, the vast majority of studies have been directed at performance in mathematical problem-solving, not at changes over time (development), and most studies have not included the executive functions of visuospatial and verbal updating, inhibition, and shifting. There is a marked need for further understanding of the direct and indirect contributions of executive functioning to mathematical problem-solving in grade 4 to extend previous research. The present study therefore takes the following into account when studying performance and development of children's mathematical problem-solving abilities: a) the specific roles of visuospatial and verbal updating, inhibition, and shifting (i.e., aspects of their executive functioning) in their mathematical problem-solving and b) the possibly mediating role of arithmetic fluency in their mathematical problem-solving. In light of what is known to date, the following research questions then arise.

1) Is children's mathematical problem-solving performance at the end of grade 4 predicted by their arithmetic fluency and executive functioning?
2) Is the association between executive functioning and development in mathematical problem-solving, if any, mediated by children's arithmetic fluency?

For the present study, a longitudinal design was adopted to monitor children's mathematical problem-solving from the start to the end of fourth grade of elementary school, with non-verbal reasoning ability controlled for as a critical factor underlying mathematical problem-solving ability (Fuchs et al., 2006).

With regard to the first research question, we hypothesized that arithmetic fluency would directly predict mathematical problem-solving when measured at the end of grade 4. Being arithmetically fluent and capable of applying factual math knowledge is clearly necessary to solve advanced mathematical problems. We also hypothesized that both visuospatial and verbal updating would directly and significantly predict mathematical problem-solving at the end of grade 4. In light of the literature, verbal updating might prove more important than visuospatial updating. The roles to be expected for inhibition and shifting were not clear but nevertheless of interest.

With regard to the second research question, we hypothesized that arithmetic fluency would mediate the associations between executive functioning and development in the children's mathematical problem-solving during fourth grade. We specifically expected both visuospatial and verbal updating to contribute to the mediating function of arithmetic fluency and thus indirectly to the development in mathematical problem-solving during grade 4 but also directly. We had no specific hypotheses about the direct influences of inhibition and shifting on development in mathematical problem-solving or possibly indirect influences via associations with arithmetic fluency. The roles of these aspects of executive cognitive functioning are nevertheless of great interest in light of the gradually more advanced mathematics presented during the fourth grade of elementary school.

The present study will pinpoint the different contributions of visuospatial and verbal updating and of inhibition and shifting (in combination with inhibition) to mathematical problem-solving in fourth grade children, along with the roles of arithmetic fluency and prior mathematical problem-solving achievement. Knowing more about the contributions of these different components may lead to more insight into how upper elementary children can be supported in their mathematics learning.

Method

Participants and study context

Participants were 458 fourth grade children from 27 mainstream elementary schools in the Netherlands. Schools were recruited via social media (Twitter) and direct mailing to the school principals and fourth grade teachers (contact information gathered via public websites from schools). Twenty-seven schools signed up to participate for two school years. Due to internal school affairs, 22 schools participated throughout the 2-year study period. The participating schools were located in rural and urban areas spread across the Netherlands and were diverse in terms of school size, pupil population, and mathematics curriculum used.

As part of a larger longitudinal research project, the data for this study were collected from a randomly selected sample of 458 out of 1062 children. This sample comprises an even distribution of low, average, and high math achieving children (based on standardized national mathematics test scores). Of the 458 children composing the sample, 50.3% was male and 49.7% female. The mean age of the children was 9;1 years (*SD* = 0.43), with a range of 8;02 to 10;10 (years; months). The spread in age was due to either having skipped a year of school or repeating a year. For 89.9% of the children Dutch was the language used in the home. Due to absences or incomplete task, performance, the amount of data collected varied from N = 388 to N = 453 per test. Only complete responding was included in the data analyses.

The Raven's Standard Progressive Matrices were administered at the start of the school year in order to check on participants' non-verbal reasoning, to ensure that none of the participants had scores 2 or more standard deviations below the mean (Raven, 2000; Raven et al., 1998). None of them did. The mean non-verbal reasoning score found for the children at the beginning of fourth grade (N = 450) was 36.58 (*SD* = 6.99), skewness -0.73, kurtosis 1.37. The sample was treated in accordance with institutional guidelines as well as APA ethical standards.

Procedure

After recruitment of participants, an information meeting was organized in two different regions of the Netherlands. During the meeting, the schools were presented both verbal and printed information about the purpose of the study, duration of the study, and data collection methods to be used. The parents of the recruited children were provided information about the study by the school. Both the schools and the parents provided their written consent for participation of the children prior to data collection.

The Cito (Dutch national standardized mathematics test) mathematics achievement data were obtained from the schools. Measures of arithmetic fluency (start grade 4) and non-verbal intelligence (start grade 4) were administered in class using paper and pencil. The children sat in a test setup so they could not copy from each other. The first author gave test instructions and stayed in the classroom. The teacher also remained in the classroom. The testing took about 45 minutes, excluding a short break between the administration of the two measures.

The executive functioning of each child (visuospatial and verbal updating, inhibition and shifting in combination with inhibition) was tested individually in a quiet room in the child's school by an educational psychologist (i.e., the first author) at the start of grade 4.

Baseline measure (start grade 4) and outcome measure (end grade 4)

Mathematical problem-solving. The children's criterion-based mathematics test performance at the end of grade 4 was adopted as the outcome measure. Standardized Dutch national tests are commonly administered at the middle and end of each school year to monitor student progress (Cito; Janssen et al., 2005). The mathematics test is made up of a mixture of computation problems (e.g., *7500 : 250 =*) and word problems. Some translated examples of word problems: *The zookeeper has 75 fish. Each penguin gets 3 fish. How many penguins can the zookeeper feed?; Elsa wants to paint the wall of her room a different color. To know how much paint she needs, she must know the surface area of the wall. The wall is 6 yards long and 2.50 yards wide. What is the surface area of the wall?* Mathematical problems are presented using mathematical notation, text, and/or text with pictures. These pictures are not just decorations but provide additional information needed to solve the problem. The majority of the mathematical problems have a picture in combination with text: *How many jars of powdered milk are in this box? ___ jars (accompanying picture depicts a full box in which only some of the jars are visible); Dad's birthday is on June 28th. He will celebrate his birthday on the following Saturday. That is on ___ (accompanying picture depicts the calendar for the month of June).*

The following mathematics domains are covered: 1) numbers, number relations, and operations (addition, subtraction, multiplication, and division); 2) proportions and fractions; and 3) measurement and geometry. The reliability coefficients for the different versions of the test (middle-end) ranged from .91 to .97 (Janssen et al., 2010), in the present study $\alpha = .86$. The test scores at the end of grade 4 were used as the outcome measure (T2); the test scores at the start of grade 4 were used as a baseline measure (T1). It must be noted that the baseline measure was actually included as part of standardized testing at the end of grade 3, but for clarity and consistency we are using this as the level at the start of grade 4. The mathematical problem-solving measure was a longitudinal measure (T1 and T2), whereas all other measures were collected before T2.

Measurement instruments

Mediator measure (start grade 4)

Arithmetic fluency. The Speeded Arithmetic Test (Tempo Test Automatiseren, TTA; De Vos, 2010) is a standardized paper-and-pencil test frequently used in Dutch education to measure speeded arithmetic skill (arithmetic fluency). The test consists of four categories of 50 fact problems: addition (tasks with a range of difficulty level from 6 + 0 to 29 + 28), subtraction (range from 4 – 2 to 84 – 38), multiplication (range from 4 x 1 to 5 x 9), and division (range from 8 : 2 to 72 : 9). Children are given 2 minutes to solve as many problems as possible within a given category. Each correct answer yields 1 point, for a total of 50 possible points per category and a total possible score of 200. The number of problems answered correctly for each category was adopted as the domain score. The total for four domains was used in the analyses. The test was administered at the start of grade 4. And the reliability and validity of testing was judged to be good (α = .88; De Vos, 2010), in the present study α = .92.

Predictor measures (start grade 4)

Visuospatial updating. The Dot Matrix and Backward Dot Matrix subtests from the Alloway Working Memory Assessment (AWMA) were used to assess so-called visuospatial updating (Alloway, 2012; Van Berkel & Van der Zwaag, 2015). The AWMA is an online assessment tool for use with children 9 to 17 years of age, the Dot Matrix is a span task that calls upon visuospatial updating. In the Dot Matrix, the child is required to watch a red dot in a sequence of locations on a four-by-four square matrix on a computer screen. The child is then asked to indicate the sequential order of locations of the red dot on a blank square on the computer screen. The number of red dots presented increases from one to nine red dots on subsequent trials and had to be recalled in the order they were presented. In the Backward Dot Matrix subtest, sets of three geometrical shapes arranged in three square frames are presented. The respondent must identify the odd-one-out shape by pointing to it and then must memorize its location (left, middle, or right). Following presentation of one or more sets of three shapes (i.e., a block composed of a minimum of one and maximum of seven sets of three shapes), the locations of the odd-one-out shapes must be recalled in the same order as presented. The subtest starts with a block containing one set of shapes and increases to a block containing seven sets of shapes. When a child made three or more mistakes within a block, the test stopped automatically. The total number of correct answers for the two AMWA subtests was used as a measure of visuospatial updating. The reliability coefficients for the Dot matrix (.83) and Backward Dot Matrix (. 82) were judged to be good in the past and also in the present study (α =.85 and .84).

Verbal updating. The Digit Span subtest from the Wechsler Intelligence Scale for Children (WISC-IV) was used to measure verbal updating (Wechsler, 2003). First, the child is asked to repeat a sequence of digits in forward order as read aloud by the examiner. The number of digits of a sequence increases from two to nine digits on subsequent trials. Then, the child is asked to repeat different sequences of digits in backward order. This task increases in difficulty from two to eight digits on subsequent trials. Every item on the Digit Span consists of two trials, each of which is scored 1 or 0 points. The test was completed when the child failed both trials of the same length. The sum of scores was calculated. Higher scores indicate better performance. The reliability coefficient for this test has found to be .88 in the past (Kaufman et al., 2006) and was .65 in the present study, which is acceptable. Forward digit span requires rote memory and auditory sequential processing while backward digit span also requires the use of working (i.e., short-term) memory for the transformation and manipulation of information.

Inhibition and shifting in combination with inhibition (shifting + inhibition). To assess inhibition and shifting, the Color Word Interference Test (CWIT) was used. This test is part of the Delis–Kaplan Executive Function System (DKEFS; Delis et al., 2001), an age-normed battery of tests designed to measure executive functions in children and adults, ages 8-89. The Color Word Interference Test (CWIT) has four conditions: Color Naming (condition 1), Word Reading (condition 2), Inhibition (condition 3), and Shifting + Inhibition (condition 4). Condition 1 involves naming the color of colored squares and condition 2 involves reading words (names of colors) aloud. Conditions 3 and 4 were used to measure inhibition and shifting. In the inhibition condition (condition 3), children must suppress a prepotent response (i.e., predisposition) by stating the color of the ink used to present a word rather than reading the word itself (which may be a color word). For example, the word ‘green’ is printed in red ink. The correct answer in this case is ‘red’, not 'green'. This task is based on the Stroop (1935) procedure. In the shifting + inhibition condition (condition 4), the child is presented with a page containing the words red, green, and blue written in red, green, or blue ink. Half of the words are presented in boxes. The respondent is asked to state the color of the ink in which the word is printed (just as in the inhibition condition) or, when the word appears within a box, instead to read the word aloud (and not name the ink color). The child has to switch between reading the word and naming the color of the ink. This must be done as quickly and accurately as possible. Each condition has two practice rows, with a total of 10 items. The 50 items were presented in five rows of 10 items each. The child has to complete each condition in a maximum of 180 seconds. When the child completed each condition in less than 180 seconds, the completion time for each condition is noted in seconds. Raw scores were used as measures for inhibition and shifting + inhibition, consisting of completion time and correct words named for each of the two conditions. For both the inhibition and shifting + inhibition conditions, faster completion times and fewer errors indicate better performance; the lower the score, the better. In the present study, the reliability for the CWIT (all four conditions) was found to be generally acceptable (.76), but questionable to acceptable for both the inhibition (.62) and shifting + inhibition (.68) conditions.

Data analyses

The data and descriptive statistics for all of the measures were first screened for potential errors and outliers. We discovered five outliers when checking for normality. We used boxplots as well as z-scores with a standard cut-off value of +/- 3.00 from 0. Outliers were then removed from the data (one non-verbal reasoning score, one inhibition, three shifting + inhibition). All of the variables were normally distributed with acceptable values of skewness and kurtosis (Field, 2009). We next computed the Pearson correlations between the predictor and outcome measures.

To address the first research question, a multiple hierarchical regression analysis was conducted with mathematical problem-solving at the end of grade 4 as the outcome variable. Arithmetic fluency and the measures of executive functioning were the independent variables.

To address the second research question, we computed mediation analyses using the process add-on Hayes, version 3.5, model 4, with a default bootstrapping at 5000 cycles (Hayes, 2018). Mathematical problem-solving at the end of grade 4 was the outcome variable. The four measures of executive functioning were the independent variables, arithmetic fluency at the start of grade 4 was a mediating variable, and mathematical problem-solving at the start of grade 4 was included as a co-variate. We estimated the direct, indirect, and total effects for each of the independent variables. The direct effects are the influence of the measures of executive functioning on mathematical problem-solving end grade 4 without inclusion of the mediator arithmetic fluency. The indirect effects are the influences of the measures of executive functioning when arithmetic fluency is included as a mediating variable. The total effect is the impact of the measures of executive functioning on mathematical problem-solving end grade 4 without inclusion of the mediator and not controlled for mathematical problem-solving performance start grade 4.

Results

Descriptive statistics

Descriptive statistics are displayed in Table 1. The correlation results showed all of the measures to correlate highly significantly with each other; see Table 2. Each of the predictor measures correlated significantly with the outcome measure. The correlations between arithmetic fluency and mathematical problem-solving were moderate. The other correlations were low but significant. Some of the correlations showed up negative, given that for some of the measures a lower score indicated better performance (e.g., inhibition and shifting + inhibition speed and number of errors).

Table 1 Descriptive Statistics

Measures	N	*M* (*SD*)	Skewness	Kurtosis
Visuospatial updating	388	19.54 (6.20)	0.01	0.11
Verbal updating	454	11.64 (2.44)	0.35	0.72
Inhibition	452	87.02 (19.73)	0.46	1.61
Shifting + inhibition	451	85.38 (18.70)	0.44	1.72
Arithmetic fluency	452	106.90 (34.49)	0.18	-0.58
Mathematical problem-solving T1	453	215.67 (28.27)	0.05	0.51
Mathematical problem-solving T2	446	239.41 (25.82)	-0.09	0.25

Note. T1 = start grade 4; T2 = end grade 4.

Table 2 Correlations Between Measures

Measures	1	2	3	4	5	6	7
1. Visuospatial updating	-						
2. Verbal updating	.218	-					
3. Inhibition	-.206	-.311	-				
4. Shifting + inhibition	-.268	-.230	.600	-			
5. Arithmetic fluency	.258	.238	-.366	-.349	-		
6. Mathematical problem-solving T1	.374	.383	-.235	-.203	.547	-	
7. Mathematical problem-solving T2	.334	.316	-.172	-.177	.490	.759	-

Note. $p < .001$ for all correlations.

3

Predicting mathematical problem-solving performance

To answer the first research question, namely whether children's mathematical problem-solving at the end of grade 4 is predicted by their arithmetic fluency and their executive functioning at the start of grade 4 (or not), the results of the multiple regression analyses were examined (see Table 3). As can be seen, 23.6% of the variance in the children's mathematical problem-solving at the end of grade 4 could be explained by their arithmetic fluency alone. When the components of executive functioning were added to the model, 31.4 % of the variance in mathematical problem-solving was accounted for. Examination of the individual contributions of the predictors in model 2 showed arithmetic fluency, visuospatial updating, and verbal updating to be significant predictors. Inhibition and shifting + inhibition at the start of grade 4 did not predict mathematical problem-solving at the end of grade 4.

Table 3 Multiple Regression Analysis for Contributions of Components of Executive Functioning and Arithmetic Fluency to Mathematical Problem-Solving at the End of Grade 4

	B	SE	ß	*t*
Model 1				
$F(1,363) = 112.310$, $p < .001$, $R^2 = .236$				
Arithmetic fluency	.362	.034	.486***	10.598
Model 2				
$F(4,360) = 33.028$, $p < .001$, $R^2 = .314$, $\Delta R^2 = .079$				
Arithmetic fluency	.311	.036	.418***	8.603
Visuospatial updating	.821	.189	.203***	4.351
Verbal updating	1.989	.493	.189***	4.032
Inhibition	.093	.075	.071	1.244
Shifting + inhibition	.010	.076	.008	.132

*Note.****p* < .001.

Predicting mathematical problem-solving development

Our second research question was whether or not any association between executive functioning and *development* (i.e., changes) in the children's mathematical problem-solving during grade 4 was mediated by children's arithmetic fluency (measured at the start of grade 4) after control for level of mathematical problem-solving at the start of grade 4. The mediation results are presented in Figure 1.

The mediation (see Figure 1) with visuospatial updating, verbal updating, inhibition, and shifting + inhibition as predictors and arithmetic fluency at the start of grade 4 as a mediator, and initial level of mathematical problem-solving as control explained 57.2% of the variance in the development of the children's mathematical problem-solving during grade 4. The indirect effects of visuospatial and verbal updating via arithmetic fluency on mathematical problem-solving at the end of grade 4 were *not* found to be significant, $a_1b_1 = .027$, 95% CI = [-.014, .084], $a_2b_2 = .043$, 95% CI = [-.064, .185]; the bootstrapped 95% confidence intervals did cover zero. Similarly, the direct effects of visuospatial and verbal updating on mathematical problem-solving at the end of grade 4 (c) were not found to be significant, $a_1b_1 = .270$, SE = .150, $t = 1.797$, $p = .073$; $a_2b_2 = .375$, SE = .359, $t = 1.044$, $p = .297$. The total effects of visuospatial and verbal updating on mathematical problem-solving at the end of grade 4 (c') were also not found to be significant, $ß_1 = .297$, SE = .151, $t = 1.962$, $p = .051$; $ß_2 = .418$, SE = .362, $t = 1.155$, $p = .249$.

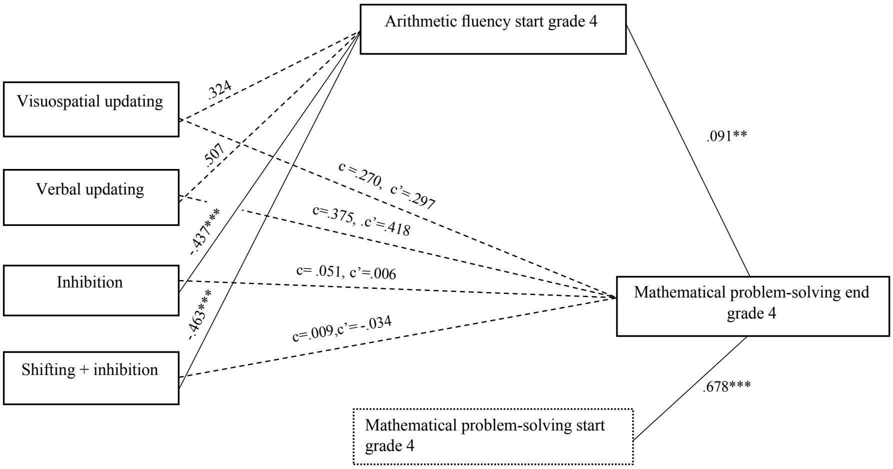


Figure 1 Results of Mediation Analyses with Measures of Executive Functioning as Predictors (at the Start of Grade 4), Arithmetic Fluency (at the Start of Grade 4) as Mediator, Mathematical Problem-Solving (at the Start of Grade 4) as Covariate, and Mathematical Problem-Solving at the End of Grade 4 as Outcome

** $p < .01$, *** $p < .001$.

In contrast, the indirect effects of inhibition and shifting + inhibition via arithmetic fluency on the children's mathematical problem-solving at the end of grade 4 were significant, a_3b_3 = -.045, 95% CI = [-.078, -.017]; a_4b_4 = -.043, 95% CI = [-.078, -.014]; the bootstrapped 95% confidence intervals did not cover zero. The direct effects of inhibition and shifting + inhibition on mathematical problem-solving (c) were *not* found to be significant, β_3 = .051, SE = .043, t = 1.172, p = .242; β_4 = .009, SE = .046, t = .196, p = .845. The total effects of inhibition and shifting + inhibition on mathematical problem-solving at the end of grade 4 (c') were also not found to be significant, β_3 = .006, SE = .042, t = .134, p = .893; β_4 = -.034, SE = .044, t = -.774, p = .439.

The association between arithmetic fluency at the start of grade 4 and mathematical problem-solving at the end grade was significant (β = .270, $p < .01$). The association between mathematical problem-solving at the start of grade 4 and mathematical problem-solving at the end of grade 4 was also significant (β = .678, $p < .001$).

In sum, visuospatial updating, verbal updating, and arithmetic fluency significantly predicted mathematical problem-solving at the end of grade 4. At least some of the development in mathematical problem-solving during the fourth grade was mediated by the children's arithmetic fluency (as measured at the start of grade 4 and after control for mathematical problem-solving at the start of grade 4). Inhibition and shifting + inhibition related directly and significantly to arithmetic fluency and therefore only indirectly with the development in the children's mathematical problem-solving during grade 4. Only arithmetic fluency directly affected the development in children's mathematical problem-solving during grade 4.

Discussion

The purpose of the present study was to identify the roles of children's arithmetic fluency and executive cognitive functioning — including visuospatial updating, verbal updating, inhibition, and shifting — in children's fourth grade mathematical problem-solving. Arithmetic fluency, visuospatial updating, and verbal updating proved predictive of mathematical problem-solving at the end of grade 4 while inhibition and shifting (in combination with inhibition) did not. With regard to the changes (i.e., development) in the children's mathematical problem-solving during fourth grade, only arithmetic fluency showed a strong and direct effect on performance at the end of grade 4 and after control for mathematical problem-solving at the start of grade 4. Inhibition and shifting (in combination with inhibition) were now found to *indirectly* relate to the children's mathematical problem-solving at the end of grade 4 *via* arithmetic fluency and to thus play a role in the development of the children's mathematical problem-solving.

Mathematical problem-solving performance

The present finding that arithmetic fluency is predictive of mathematical problem-solving at the end of grade 4 is consistent with previous findings (Fuchs et al., 2006; Viterbori et al., 2017; Zheng et al., 2011). Being arithmetically fluent and thus able to quickly access and apply factual knowledge is clearly necessary for the solution of advanced mathematical problems. Of the components of executive cognitive functioning, visuospatial updating and verbal updating were predictive for the children's mathematical problem-solving at the end of grade 4, inhibition and shifting (in combination with inhibition) were not. This finding is also consistent with the findings of previous studies showing principal roles for visuospatial and verbal updating in mathematical problem-solving (e.g., Andersson, 2007; Cragg et al., 2017; Passolunghi & Pazzaglia, 2004; Zheng et al., 2011). Indeed, mathematical problems with more abstract and predominantly verbal information are increasingly presented in grade 4. Verbal updating gains importance, in addition to visuospatial updating (Andersson, 2007; Van de Weijer-Bergsma et al., 2015). It should be noted that we did not have specific expectations about the possible contributions of inhibition and shifting (in combination with inhibition) to the prediction of the fourth grade children's mathematical problem-solving and did not find significant contributions. In a meta-analysis of previous studies that included visuospatial updating, verbal updating, inhibition, and shifting to examine children's mathematical problem-solving, the executive functions of visuospatial updating and verbal updating were also found to predominate — just as in the present study — over inhibition and shifting in the prediction of mathematical problem-solving (Friso-van den Bos et al., 2013).

Mathematical problem-solving development

With regard to the changes/development in the children's mathematical problem-solving during grade 4, we hypothesized — on the basis of a more recent study by Fuchs et al. (2016) — that starting arithmetic fluency would mediate any associations between the executive functioning of the children and changes in their mathematical problem-solving. This was indeed found to be the case. Unexpectedly, however, the executive functions of inhibition and shifting (in combination with inhibition) as opposed to visuospatial updating and verbal updating were found to indirectly contribute to mathematical problem-solving at the end of grade 4 via starting arithmetic fluency and after control for the children's mathematical problem-solving at the start of grade 4. Declining importance for visuospatial updating and verbal updating has also been found in a few other studies when mastery of the relevant mathematical content within a given domain can be assumed to have increased (e.g., mastery of basic arithmetic in grade 4; Balhinez & Shaul, 2019; Fuchs et al., 2006). In the present study, we nevertheless expected both visuospatial and verbal updating to continue to play both direct and indirect roles in the changes/development of children's mathematical problem-solving during grade 4, which did not prove to be the case. The finding of significant roles for inhibition and shifting + inhibition was unexpected. The children in our study had to solve increasingly more advanced, multi-step mathematical fact and word problems, with/without pictures, requiring a variety of calculations within a single problem. To solve such multiple step problems, inhibition and shifting may be more critical than visuospatial and verbal updating (Bull & Scerif, 2001; Cantin et al., 2016; Verschaffel et al., 2020). For example, when children confront a new domain of mathematics entailing increasingly complex and abstract mathematical problems, inhibition may be increasingly needed to suppress irrelevant information (e.g., irrelevant textual information) and prior learning experiences (e.g., ignoring a counting on strategy when applying a multiplication strategy is more appropriate). In addition, shifting is increasingly needed to switch between procedures (e.g., going from addition to multiplication, shift to another strategy; Wiley & Jarosz, 2012). At this point in the child's learning then, visuospatial and verbal updating may still be important but not as important as when the child is less arithmetically fluent. In other words, the roles of inhibition and shifting in mathematical problem-solving may increase in grade 4 but remain indirect as they still depend on arithmetic fluency (Cragg et al., 2017). As children learn to solve a wider variety of mathematical problems in grade 4, greater flexibility in the determination of solution strategies and conduct of calculations is needed (Fuchs et al., 2006; 2016; Geary, 2011; Wiley & Jarosz, 2012). The executive function of inhibition and/or shifting comes to play an increasingly important role in children's mathematical problem-solving as found in the present study.

Finally, the results of the present study indicate that while the level of mathematical problem-solving at the start of grade 4 is predictive for the development of mathematical problem-solving ability (and therefore used as a control variable in some of our analyses), the level of arithmetic fluency is equally important and continues to be important. These findings are in line with the hierarchical frameworks for understanding changes in children's mathematics achievement over time and the assumption that the influences of various aspects of children's executive functioning are mediated during their development by the concomitant development of domain-specific mathematical competencies (Cragg et al., 2017; Geary, 2004; Geary & Hoard, 2005).

Study strengths, limitations, and directions for future research

A major strength of the present study is the large and representative sample size of 458 children from 27 elementary schools, with also control for the children's non-verbal reasoning capacities. Also, a strength of the study is the use of children from grade 4 or, in other words, children facing the challenge of solving increasingly complex and more abstract mathematical problems but also expanding their knowledge and skills to include new domains of mathematics. Direct measures of executive functioning were used and important aspects of executive functioning were distinguished in doing this: visuospatial updating, verbal updating, inhibition, and shifting (in combination with inhibition). Two mathematics tests that have been proven to be reliable were also used: one for arithmetic fluency and one for more advanced fact and contextual mathematical problem-solving.

The present study also has some possible limitations. Multiple measures were not used to assess the four components of executive functioning, although doing this might have yielded more reliable results (e.g., use of two different tests per executive function, use of a measure that focuses exclusively on shifting). Furthermore, for follow-up research, we recommend including the measurement of arithmetic fluency at the end of grade 4 and using a structural equation model to examine the direct and indirect effects over time, in a cross-lagged design. In addition, we did not explore just how the children went about solving the mathematical problems presented to them. Observational methods might therefore be incorporated into future studies to provide a process measure of children's mathematical problem-solving. By doing this, for example, Kotsopoulos and Lee (2009) found that executive updating (with no distinction between visuospatial and verbal updating) was most challenging during the phase of understanding a mathematical problem, inhibition during the planning phase, and shifting during the reflection/evaluation phase. Another possible limitation on the present study is that other potentially relevant factors — such as children's reading comprehension, task approach, and (in) adequate identification of problem-solving strategies — were not included. Consideration of these factors in future research is therefore recommended.

Implications for Practice

Solid mastery of starting mathematical knowledge and skills obviously facilitates later learning and mathematical problem-solving (Watts et al., 2014). Careful attention should therefore be paid in the teaching of mathematics to the establishment of a solid mathematical foundation during the early elementary school years. Children with poor arithmetic fluency especially require explicit instruction and intensive training to improve their arithmetical knowledge and efficient strategy use (Koponen et al., 2018). Children need arithmetic fluency and sufficient prior mathematical knowledge for successful mathematics learning in grade 4 and subsequent grades.

With regard to executive functions, attempts to improve executive functioning have shown limited transfer to other domains and long-term effect from interventions are largely unknown (Diamond, 2012). Based on a recent study by Gunzenhauser and Nückles (2021), supporting executive functioning during daily mathematics lessons in several ways can be suggested. One suggestion is modeling by the teacher; that is, the teacher can demonstrate how to make a plan and monitor its implementation in solving a complex mathematical problem. Another suggestion is informed training; that is, the teacher provides information about how, when and why to enact a particular skill. Furthermore, it is important that teachers consider the specific executive functions that might help children to solve mathematical problems and scaffold the children during instruction (e.g., break complex problems into manageable parts, teach strategies to deal with irrelevant information).

Conclusion

The present research findings provide further insight into the roles of arithmetic fluency and specific aspects of executive functioning in the mathematical problem-solving of children. Arithmetic fluency and the visuospatial and verbal updating aspects of executive functioning appear to be most important for mathematical problem-solving measured at the end of grade 4. When mathematical problem-solving measured at the start of grade 4 is controlled for and the *development* in children's mathematical problem-solving during grade 4 is considered, the executive functions of inhibition and shifting (in combination with inhibition) are now seen to directly relate to arithmetic fluency and indirectly to development in mathematical problem-solving. An important finding in this study is the continued and unique contribution of arithmetic fluency to the mathematical problem-solving of children in grade 4, which required a more advanced level of mathematical problem-solving than in previous studies using younger children.

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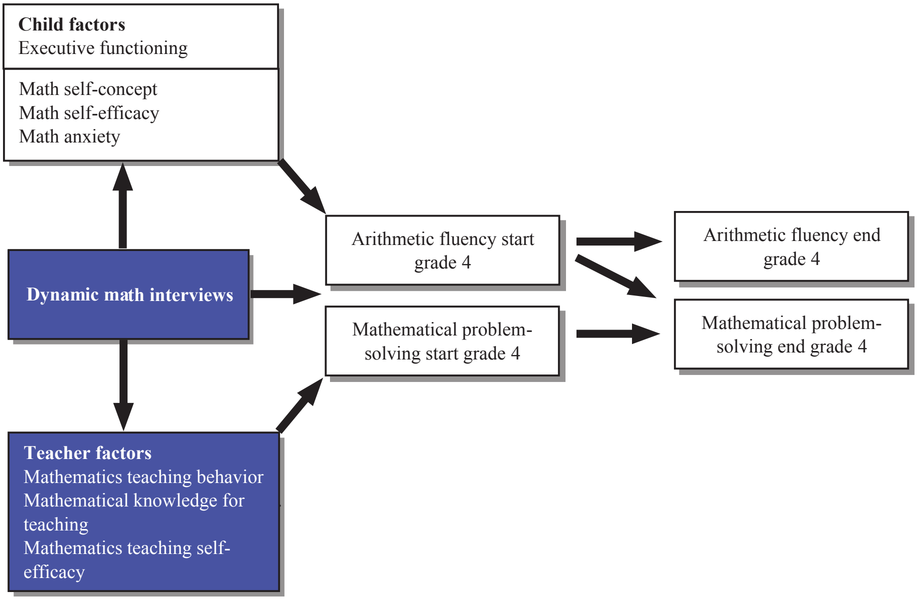
Chapter 4

Effect of dynamic math interviews on mathematics teaching

A manuscript, based on this chapter, has been submitted for publication.

Abstract

In this quasi-experimental study involving 23 fourth grade teachers, we investigated the effect of implementing teacher-child dynamic math interviews to improve mathematics teaching behavior in the classroom. After a baseline period of 13 months, 23 fourth grade teachers were given dynamic math interview intervention which consisted of a teacher professional development program followed by a period of practice in math interviewing to identify children's math learning needs. To determine the effects of the teacher professional development program, pretest and posttest videos of math interviews were compared. To analyze the effects of the intervention, mathematics teaching behavior, perceived mathematics teaching self-efficacy and mathematical knowledge for teaching were measured. Results showed not only the effect the program had on the quality of the dynamic math interviews, but also the effects of the intervention on mathematics teaching behavior, mathematics teaching self-efficacy and mathematical knowledge for teaching. Dynamic math interviews can be used to improve mathematics teaching practice.



Introduction

The premise of mathematics education is that teachers, through the use of effective mathematics teaching practice, can offer all children the opportunity to maximize their learning potential (Forgasz & Cheeseman, 2015). Achieving this requires understanding the diverse learning needs of all children and the ability to adapt to these needs in the regular mathematics classroom setting (Forgasz & Cheeseman, 2015). Meeting a variety of children's needs is complex and a major challenge for many mainstream teachers. Teachers must be able to handle multiple learning trajectories and provide tailored support to learners of different math abilities (Deunk et al., 2018). In order to adjust their teaching, teachers must be able to identify children's math learning needs. Dynamic math interviews may be able to help identify these needs (Allsopp et al., 2008; Ginsburg, 2009). Dynamic math interviewing is a flexible, semi-structured mathematics assessment approach in which the teacher interacts with a child to get insight into their mathematical thinking, conceptual understanding and underlying procedures and strategies as well as their beliefs and emotions regarding mathematics (Allsopp et al., 2008; Ginsburg, 2009).

In order to meet the different educational needs of diverse learners, three teacher factors have been identified as essential (Kaiser et al., 2017). The first is effective teaching behavior during mathematics lessons (e.g., Anthony & Walshaw, 2009). The second is teachers' self-efficacy regarding mathematics teaching, i.e., their beliefs in own capabilities to influence child's learning, achievement and engagement (e.g., Chang, 2015). The third is teachers' mathematical knowledge for teaching, including deep knowledge of content and the knowledge and skills specific to teaching mathematics (e.g., Hill et al., 2008).

The current study investigated whether a teacher professional development program focused on dynamic math interviews helps teachers better execute such interviews with respect to identifying children's math learning needs. Furthermore, it has yet to be established whether teacher-child dynamic math interviews are related to other classroom teaching factors such as mathematics teaching behavior, teachers' sense of mathematics teaching self-efficacy and teachers' beliefs in their mathematical knowledge for teaching.

Dynamic math interviews as cornerstone to identify child needs and foster mathematics teaching

The need for teachers to measure the developmental potential of children – not only their present ability – has led to dynamic mathematics assessment approaches closely linked to contemporary conceptions of learning and mathematics education (e.g., Ginsburg, 2009; Jeltova et al., 2007). Pellegrino et al. (2001) designed a learning assessment model consisting of three elements that needed to be connected by the teacher. The first was the way in which children represented knowledge and developed subject domain competence (cognition). The second involved teachers observing children's performance (observation). The third required teachers to analyze data based on child interactions with specific domain tasks (interpretation). A dynamic mathematics assessment fulfills all these requirements. It is flexible and process-oriented and enables teachers to obtain information about diverse children' thinking and conceptual understanding across the mathematic curriculum (Allsopp et al., 2008). It can provide more insight into children's mathematics learning capabilities than traditional tests (Seethaler et al., 2012).

A dynamic math interview is a dynamic assessment approach in the form of a semi-structured diagnostic interview where teachers conduct process research in the various domains of mathematics. In such an interview, teachers can assess achievement levels, underlying procedures and strategies, and the type of support children need for further mathematical development (Ginsburg, 1997; Van Luit, 2019). The formative information teachers gather from dynamic math interviews could be used to develop micro-interventions in the classroom including, for example, use of representations, additional instruction, offering challenging and engaging tasks. These interventions, in the zone of proximal development of children, support children's learning and problem-solving abilities and promote child's self-esteem (Deunk et al., 2018).

When interacting with children, teachers stimulate child responses, can better understand their points of view and help address specific educational needs (Lee & Johnston-Wilder, 2013). Teachers can communicate in a way that helps the child discover his or her mathematics learning strengths, experiences and emotions towards mathematics learning and goals and the support needed to achieve them -- linked to the future-focused solution-focused approach (Bannink, 2010). In a review study of applications of the solution-focused approach with children in school settings, Kim and Franklin (2009) found that this approach reduced the intensity of negative feelings and led to improved academic outcomes.

To successfully identify and adapt to children' math learning needs, teachers need insight into their mathematical performance, thinking, understanding, and beliefs (Deunk et al., 2018). Dynamic math interviews may be an effective tool to help gain these insights (Ginsburg, 1997, 2009). To our knowledge, only few scripted tools for mathematics assessment exist and these focus on specific domains of mathematics (Emerson & Babtie, 2014; Wright et al., 2006).

Teacher factors regarding mathematics teaching

Teachers' ability to create opportunities for all children and adjust their teaching to meet a variety of additional support needs is the cornerstone of mathematics teaching (Forgasz & Cheeseman, 2015). In order to meet the different educational needs of diverse learners in a mathematics classroom context, teachers must be competent in coping with a diversity of learning trajectories, and providing support during learning. For mathematics teaching to be successful, teachers need an informed view of children' understanding of mathematics and their educational math needs (Deunk et al., 2018). Therefore, teachers need pedagogical, didactical and subject knowledge and the ability to effectively apply this knowledge within mathematics lessons. This might, for example, involve create a sequence of tasks or drawing a model or diagram and encouraging children to explain their strategy. Teachers also need insight into their children' current mathematical thinking as well as tools and strategies for representing and explaining mathematics that are in line with children' educational needs (Reynolds & Muijs, 1999). Dynamic math interviews can help provide this insight (Allsopp et al., 2008). Such insight refers to the ability to make efficacious decisions regarding child-related instructional goals, to master relevant prior knowledge and skills within several mathematical domains, to recognize children's preconceptions or misconceptions, to assess children's motivation and to group and support children according to ability (Hoth et al., 2016). Teachers who pursue effective mathematics teaching appear to use some general structural aspects of differentiation, such as achievement grouping combined with differentiation (Prast et al., 2018). Deunk et al. (2018) showed that teachers found it challenging to provide refined adaptations that met an individual child's math learning needs.

Various teacher factors influence mathematics teaching. To gain comprehensive insight into professional mathematics teaching abilities, Kaiser et al. (2017) posited three keys to successful mathematics teaching: mathematics teaching behavior, self-efficacy in regard to teaching mathematics and mathematical knowledge for teaching.

With respect to effective teaching behavior, Reynolds and Muijs (1999) found that classroom management, the ability to teach math concepts while correcting misconceptions, interactive and activating teaching and providing adjusted support were important predictors of children's mathematics achievement. In addition, based on observing elementary school mathematics lessons, Van de Grift (2007) identified the following as variables affecting the quality of teaching: a safe and stimulating learning climate, efficient classroom management, clear instruction, activating learning, adaptive teaching, and teaching and learning strategies (e.g., model, explain, scaffold). Follow-up research found that a cumulative organization of complexity levels in teaching behavior was relevant. These ranged from the less complex, such as safe learning climate and efficient classroom management, to the more complex, such as learning strategies and differentiation and adapting lessons (Van der Lans et al., 2018). By using cumulative organization, observers were able to assess teachers according to levels of complexity. And teachers could better understand their effectiveness at each level and anticipate teaching needs at the next level. Teachers' diagnostic skills were also found to be important in identifying children' math learning needs. Teachers who were able to assess children' mathematics achievement and learning and thinking processes provided better adapted student support in the classroom (Hoth et al., 2016; Ketterlin-Geller & Yovanoff, 2009).

Self-efficacy refers to teachers' perceptions of abilities about teaching and is context-specific (Tschannen-Moran & Woolfolk Hoy, 2001). In regard to mathematics teaching, several studies showed that teachers' mathematics teaching self-efficacy influence children's learning, achievement and engagement. Chang (2015) found that teachers' mathematics teaching self-efficacy influenced children's self-efficacy and mathematics achievement as well. On the one hand, successful mathematics teaching acquirements contributes to strong beliefs in teachers' self-efficacy and, on the other hand, children's successful learning acquirements are influenced by their teacher's effective teaching performance, which is strengthened, in part, by the teacher's self-efficacy beliefs. Furthermore, Nurlu (2015) showed that teachers with higher mathematics teaching self-efficacy beliefs took greater responsibility for children' successes and failures and made more effort to support low-achieving children. A lower sense of efficacy beliefs had a negative impact.

With reference to mathematical knowledge for teaching, a distinction can be made between pedagogical content knowledge (knowledge of content and student, and knowledge of content and teaching, e.g., teacher's ability to select and use representations and models) and subject matter knowledge (e.g., understanding concepts, skills, symbolism, procedures and student errors). A subdomain of subject matter knowledge is specialized content knowledge specific to teaching mathematics, including selecting good examples, representations, models and explanations for adapting instruction and asking questions to elicit children's mathematical thinking (Ball et al., 2008). Teachers' mathematical knowledge for teaching has been related to the quality of mathematics teaching -- especially instructional quality (Hill et al., 2008). However, other research has suggested that this relationship is nuanced rather than clear cut. According to Wilkins (2008), many variables appear to play a role in mathematics teaching practice, including beliefs and attitudes towards teaching mathematics. Charalambous (2015) found that mathematical knowledge for teaching and teachers' beliefs about teaching and learning mathematics interact to inform teaching behavior. In other words, mathematical knowledge for teaching is not enough to ensure successful teaching.

To summarize, research established three key components of effective mathematics teaching and these factors can help teachers better meet individual children's math learning needs. To successfully identify these needs and adapt to them, teachers require insight into children's mathematics performance, thinking, understanding, emotions and beliefs. Dynamic math interviews may be an effective way to help gain these necessary insights. However, the ability to conduct dynamic math interviews requires specific knowledge and skills. A teacher professional development program that encourages the development of necessary competencies could help (Heck et al., 2019).

Effective teacher professional development

Teachers can benefit from professional development programs (Jacob et al., 2017). Literature identifies the following characteristics as able to effectively influence teacher professional development: active and practice-based learning, collective participation, focus on content and classroom practice, collaboration, duration and coherence (Desimone, 2009; Heck et al., 2019; Van Driel et al., 2012). Using selected video clips from mathematics lessons in teacher mathematics training is also effective (Borko et al., 2011). Tripp and Rich (2012) explored how video influenced teacher change. They found that video and discussion motived and helped teachers adjust their teaching. Their work showed that teachers rated video analyses as a very effective feedback tool. In addition, Heck et al. (2019) found that a teacher mathematics training which strengthens connections between the development of mathematics teaching behavior, mathematics teaching self-efficacy and mathematical knowledge for teaching, is effective.

To our knowledge, no study has examined a teacher professional development program focused on the knowledge and skills needed to conduct teacher-child dynamic math interviews. Scripted tools could help the teachers conduct dynamic interviews, but these are few and far between (Caffrey et al., 2008). For the purpose of the present study we created a tool to enhance teacher-child dynamic math interviews to identify math learning needs that was based on relevant research (Allsopp et al., 2008; Bannink, 2010; Delfos, 2001; Ginsburg, 2009; Ketterlin-Geller & Yovanoff, 2009). This tool enables the teacher to conduct a process-oriented math interview for various domains of mathematics and to examine math-related experiences, emotions and beliefs of the children. For example, the tool offers suggestions for questions that help gain insight into what the child understands, questions that can support the child's thinking about solutions and future goals as well as suggestions for providing support. In this way, the tool goes beyond standardized norm-referenced testing and existing assessment tools (Allsopp et al., 2008; Franke et al., 2001; Wright et al., 2006).

To date, few studies have investigated the effects of teacher professional development on teacher factors within the context of mathematics teaching in elementary schools (e.g., Jacob et al., 2017). However, the effects of professional development about dynamic math interviews on teachers' mathematics teaching behavior and perceived mathematics teaching self-efficacy and mathematical knowledge for teaching, have not yet been studied.

The present study

Dynamic math interviews may be a promising tool in the development of mathematics teaching skills. However, the direct link between teacher professional development focused on dynamic math interviews and successful, interactive teacher-child interviews is not clear. We still do not know if dynamic math interviews actually improve mathematics teaching ability. Therefore, the current study examines the extent to which teachers can be trained to give teacher-child dynamic math interviews that help identify the math learning needs of fourth grade children in the Netherlands. The study also explores whether these interviews can improve mathematics teaching. This intervention study was designed to answer the following questions:

1. What is the effect of a teacher professional development program in teacher-child dynamic math interviewing on the quality of the dynamic math interviews with fourth grade children?
2. What is the effect of teacher-child dynamic math interviews on factors of mathematics teaching (teaching behavior, self-efficacy, and mathematical knowledge for teaching)?

To answer the first question, 23 fourth grade teachers were enrolled in a teacher professional development program focused on knowledge and skills related to dynamic math interviewing. The program was followed by a practice period focusing on the ability to use dynamic math interviews to identify children's educational needs when learning mathematics. We expected that the professional development program, based on effective characteristics of teacher professional development, would have a positive effect on the quality of dynamic math interviews.

To answer the second research question, teacher factors were measured on four occasions. Three baseline measurements were taken before and one after the intervention period. We expected that the teacher professional development program would have an effect on all three teacher factors regarding mathematics teaching. We thought that identifying children's math learning needs and making the transfer to daily educational practice might appeal to teachers' specific mathematical knowledge, skills (e.g., ask appropriate questions, make appropriate interventions) and belief in their own capabilities. Experiencing dynamic math interviews and gaining insight into children' knowledge and thinking may support effective mathematics teaching.

Methods

Participants and study context

This study was undertaken within the context of the Dutch primary education system which seeks to provide appropriate education to all children. Participants were recruited by open invitations via social media (Twitter) and by direct mail addressed to both elementary school principals and fourth grade teachers (contact information gathered via schools' public websites). Interested teachers were invited to an information meeting to learn about the aims of the study, what was expected from participants, and what they could expect from the researchers. Thirty-one teachers, responsible for teaching 610 nine year old elementary school children in grade 4 (children aged 8-10 years), were initially involved. Due to illness, pregnancy, job changes and other factors, 23 teachers responsible for teaching 452 elementary school children, completed the two-year study. In the first year, measurements were taken but no interventions took place. The participants came from 22 Dutch elementary schools and had an average of 12,8 years of experience (SD = 9.8) (range of 3 to 40 years) each. Most of the teachers (70%) had a bachelor's degree in education. An additional 26% had some graduate training and 4% had a master's degree in education.

Each group of children was divided into three mathematics levels. Children were classified according to the results of the criterion-based standardized national Dutch mathematics tests given at the end of grade 3 (about 9 year old). These tests, designed to monitor math progress, are given twice a year (Janssen et al., 2005). Children at every mathematics level have educational needs. Therefore, the researchers randomly selected three low math achieving, three average math achieving and three high math achieving children per group to make sure all mathematics levels were represented. Teachers were asked to conduct dynamic math interviews with three children in their group during professional development and with six from the selected children during practice sessions. The sample was treated in accordance with institutional guidelines as well as APA ethical standards. Schools, parents, and children were informed about the procedures, duration and purpose of the research. They were also given the name of a contact in the event they had additional questions. Both schools and parents gave active, informed participation consent.

Design

To obtain a robust baseline, measurements of teacher factors were taken on three occasions -- the start and end of the first school year and the beginning of the second school year (T1, T2, T3). The fourth measurement was taken after the intervention period, at the end of the second school year (T4). The 2-year research project design is shown in Figure 1. In this quasi-experimental design, all teachers were followed in their school setting for two years and all teachers underwent the same procedure.

The effect of the dynamic math interview teacher professional development program was measured via a pretest-posttest design. The *intervention* consisted of the professional development program and a practice period where each participating teacher conducted dynamic math interviews with six children at different mathematics achievement levels. The effect was determined by comparing teacher factors regarding mathematics teaching before and after the intervention.

School year 1							
Aug-Sep	Oct	Nov-mid Feb			Feb	March-mid June	June
Measurement T1, year 1	Mathematics taught as usual						Measurement T2, year 1
School year 2							
Measurement T3, year 2	*Individual feedback on a conducted dynamic math interview*	Pre test	Teacher Professional development program	Post test	*Individual feedback on a conducted dynamic math interview*	Practice period	Measurement T4, year 2

Figure 1 The research design
Note: T1, T2, T3 = Baseline.

Intervention

The intervention consisted of a teacher professional development program comprising four, 4-hour meetings, followed by a period of dynamic math interview practice. The teacher professional development program was based on the design features of professional development (e.g., Desimone, 2009; Heck et al., 2019; Van Driel et al., 2012). These features are the collective participation of teachers of the same subject (mathematics) and grade (4), employing active and useful learning activities (e.g., good practices of math interviews), focus on content (related to dynamic math interviews and mathematics teaching), focus on inclusive mathematics classroom practice (coping with different needs of mathematics learners), collaboration (e.g., discussing articles, watching each other's math interview videos and giving peer feedback), coherence (e.g., identifying the needs of the teachers prior to the professional development program using the same math interview tool) and generous time investment.

The program's design prototype was reviewed by five students enrolled in professional Educational Needs in mathematics/dyscalculia Masters programs, one school coordinator of mathematics and one researcher in mathematics education. The review occurred between May and June at the end of the first school year. The teacher professional development program was fine-tuned in August and September, at the beginning of the second school year. The teacher professional development program included an explanation of the tool for dynamic math interviews and mathematical knowledge for teaching related to dynamic math interviews, examples of dynamic math interviews on video, and peer feedback in the second and third meeting. The first author, an expert teacher trainer, organized the meetings.

The 4-hour meetings began a few weeks after the pretest. Between the first and the second, and the second and the third meeting, the teachers practiced giving a dynamic math interview. This was videotaped. In the subsequent meeting, teachers, divided into groups of two or three, provided qualitative peer feedback. They did this using a theory-based observation tool specifically constructed for the study (e.g., kinds of questions, focus of the questions, types of support). In this way, teachers received structured peer feedback on their performances in the second and third meeting.

Teachers undertook the posttest – a video-recorded dynamic math interview – between the third and fourth meeting. It was submitted at the fourth meeting. At that last meeting's end, teachers filled in a written evaluation form about the training. With respect to content and achieved goals, the average teacher training satisfaction score was 3.55 on a scale of 1 to 4.

Each teacher received individual pretest feedback from the researcher before the first and posttest feedback after the last meeting. In the practice period that began after the four meetings, the teachers undertook and recorded another dynamic math interview with six of the nine selected children in their group. Teachers were not given feedback on these interviews; they provided proof that the dynamic math interviews actually took place (i.e., treatment fidelity).

Measurement instruments

Pretest and posttest dynamic math interview

Pretesting and posttesting consisted of a video-recorded assignment: teachers conducted a dynamic math interview in which three teacher-selected word math problems (presented using mathematical notation, text, and/or pictures) were assessed. These problems were culled from criterion-based standardized Dutch national mathematics tests

(Janssen et al., 2005). The teachers received the same instruction before the pretest and posttest. They were asked to conduct a math interview in a fashion they considered adequate.

We developed a theory-based coding book to analyze pretest and posttest transcripts based on Mayring's qualitative content analysis (Mayring, 2015). The following nine aspects of dynamic assessment that contribute to the quality and effectiveness of a dynamic math interview were analyzed. The total number of questions per aspect was counted.

1) *Questions focused on child's math experiences, beliefs and emotions.* The teacher can ask questions that widen the scope of the dynamic math interview, such as 'What do you like the most about mathematics lessons? What kinds of mathematical problems do you find hard? How does it feel when you successfully solve a problem?' (Allsopp et al., 2008; Bannink, 2010; Ginsburg, 1997).
2) *Questions focused on child's thinking and solving processes*. The teacher can pose process-oriented questions such as 'How did you solve this mathematical problem? Tell me.' (e.g., Allsopp et al., 2008; Ginsburg, 1997, 2009).
3) *Questions to identify a) child's mathematics needs in general, with active input of the child's own voice b) child's instructional needs and c) child's needs regarding content and methods.* The teacher can ask questions to identify child's math learning needs, such as 'What do you need to practice the multiplication tables?' The questions could have a solution-focused character designed to elicit student's voice, e.g., 'What is your next aim in learning mathematics? What do you need to reach that goal? How can you be helped to solve these mathematical problems?' (Bannink, 2010; Lee & Johnston-Wilder, 2013).
4) *Questions to check whether the child knows the right answer*. These questions are product-oriented, designed to assess the correctness of the child's answer. Although correctness of answers is important, obtaining process information must prevail for the reason that standardized tests even though focused at products (Franke et al., 2001).

4

5) *Questions to determine the level and adequacy of child's prior knowledge and understanding.* The teacher can ask qualitative and quantitative questions to gauge child's prior knowledge and understanding of mathematics concepts and procedures (Van Luit, 2019). For example, the teacher checks the procedural knowledge of division tasks while assessing the domain of fractions.
6) *Give support.* The math interview tool contains suggestions on ways the teacher can support for child's thinking and solution processes. These include giving support a) by structuring, b) by reducing complexity, c) verbally (e.g., hints), d) by using representations of real situations, e) by using models or schemes, f) by using concrete materials, g) by modelling. Some suggestions for support were developed by Gal'perin based on Vygotksy's action theory (Gal'perin, 1978); others are based on Van Luit (2019).
7) *Safe and stimulating climate during math interview.* In order to conduct a good math interview, several conditions must be met. These include a relaxed and warm atmosphere, respect, starting with a mathematical problem the child is likely to solve, verbal encouragement and sincere, supportive remarks (Delfos, 2001).
8) *Teacher summarizes the math learning needs.* The teacher succinctly summarizes child's needs using the child's own words. In this way, the teacher shows that he/she has been listening attentively and can confirm the educational needs and goals. This fosters co-responsibility by both the teacher and the child (Delfos, 2001; Bannink, 2010).
9) *Scope of the dynamic math interview*. A narrow scope meant that the math interview was aimed at obtaining information about a limited number of aspects of the child's mathematical development. A wide scope is focused on more aspects and is therefore preferred (Ginsburg, 1997).

The coding book was improved and refined based on feedback from five mathematics teaching experts (one validation sessions) and eight researchers (one validation session). For the purpose of this study, the coding book encompassed nine aspects that contribute to the quality and effectiveness of a dynamic math interview.

All pretest and posttest videos, which varied in length, were fully transcribed. We sought to compare segments of equal length using Dudley's approach (2013): we analyzed two segments, 5 minutes in total, from each video. These segments included 2 minutes taken at the beginning of the interview (0.30-2.30) and 3 minutes later on. The second segment showed the child solving three mathematical word problems which were selected, beforehand, by the teacher. The first author analyzed the pretest and posttest videos using the validated coding book.

A mathematics teaching expert with a university master's degree in special education – blind to the research design and scope – analyzed randomly selected transcripts using the same coding book. The inter-rater reliability for scoring was good (0.93).

Teacher factors

Actual teaching behavior in mathematics lessons. The actual mathematics teaching behavior was measured using an observation instrument: The International Comparative Analysis of Learning and Teaching (ICALT, Van de Grift, 2007). ICALT looks at a broad range of teaching behavior, but is not math-specific. Therefore, in this study, ICALT was supplemented with other tools specifically addressing mathematics teaching. ICALT examines 32 factors across six scales of teaching behavior; a seventh scale focuses on children's involvement. The teacher behavior scales range from lower to higher order teaching behavior: providing a safe and stimulating learning climate, efficient classroom management, clarity of instruction, activating learning, teaching learning strategies, and differentiation and adapting lesson (Van der Lans et al., 2018). Because the ICALT is not math-specific, we developed an additional eighth scale incorporating eight aspects of math-specific teaching strategies for this study (based on Gal'perin, 1978 and Polya, 1957) They are 1) informal manipulation, 2) representations of real objects and situations, 3) abstract mental representations (models and diagrams), 4) abstract concepts/mental operations, 5) making connections between the previous four levels and using these connections to support lesson goals, 6) focus on planning, 7) problem-solving processes, and 8) metacognitive skills (see Appendix A). The internal consistency of the ICALT and the supplemental scale (ICALT+S) used in the present study was good at all four timepoints (range of α 0.85-0.86). The internal consistency of all subscales in the study was also good (range of α 0.85 and higher).

Teachers' self-efficacy. The Dutch online version (Goei & Schipper, 2016) of the long form of the Teachers' Sense of Self Efficacy Scale (TSES; Tschannen-Moran, & Woolfolk-Hoy, 2001) was used to measure teachers' self-efficacy with respect to the teaching of mathematics. The questionnaire contained 24 items divided among three subscales. They were efficacy for student engagement, efficacy for instructional strategies and efficacy for classroom management. The teachers responded to a 9-point scale ranging from 1 (= *not at all*) to 9 (= *a great deal*). The reliability of the study was good. The Cronbach's alpha score was 0.86 at all four timepoints.

Teachers' mathematical knowledge for teaching. Teachers' beliefs in their mathematical knowledge were measured with a Teachers' Sense of Mathematical Knowledge for Teaching Questionnaire (TSMKTQ; Kaskens et al., 2016; see Appendix B)) – an online questionnaire developed for the current study. The 38 questions were focused on teachers' pedagogical content knowledge, subject matter knowledge or specialized content knowledge. Teachers rated all items of the questionnaire on a 4-point response scale, ranging from 1 (= *to a very small extent*) to 4 (= *to a very large extent*). The internal consistency of the TSMKTQ was good. The Cronbach's alpha was 0.86 at all four timepoints measured.

Procedure

After participants were recruited, an information meeting was organized in two regions of the Netherlands. The teachers were given printed information about the study and a fact sheet about the data collection methods to be used. The teachers gave email consent to be observed and video-recorded giving a mathematics lesson. As part of the larger, longitudinal research project (Kaskens et al., 2020), teacher competency data were obtained at four measurement timepoints (see Figure 1).

The pretests and posttests were comprised of video-recordings – one taken before and a second, at the end, of the teacher professional development program. These video-recordings were handed in to the researcher.

To assess mathematics teaching behavior, mathematics lessons given by each teacher were observed and video-recorded on four occasions. The lesson topic – either fractions or proportions – was determined in advance. The video-recordings were scored using ICALT+S observations consisting of 40 items with 4-point Likert scales ranging from 1 (= *predominantly weak*) to 4 (= *predominantly strong*). The scoring of full lessons was done by the first author and a second observer, both trained and certified to use ICALT. The inter-rater reliability for live scoring was good (0.86).

The TSES and TSMKTQ were sent to the participating teachers by e-mail. A direct link was embedded in the web-based questionnaire services in Formdesk. Teachers were asked to complete the questionnaires at the beginning and at the end of each school year; reminders were sent on each occasion. The response rate was 100% and all data collected from all 23 teachers was included in the subsequent analyses.

Data analyses

To address the first research question, the effect of a teacher professional development program on the quality of dynamic math interviews, we conducted paired samples *t*-tests (2-tailed). We controlled for multiple testing using the Bonferroni correction (see Table 1).

We checked normality using Shapiro Wilk, which is more appropriate for small sample sizes. The following outliers were computed: questions to identify children' instructional needs, questions to identify children' content and method needs, reduced complexity support, verbal support, material support and support using representations. Because the final analysis was not affected by inclusion or exclusion of these data, we left it in.

To examine the effect of the intervention on teacher factors, research question 2, we conducted three repeated-measures ANOVAs– actual mathematics teaching behavior and teachers' perceived mathematics teaching self-efficacy and mathematical knowledge for teaching – to compare differences between timepoints (baseline T1, T2, T3 and T3-T4, before and after the intervention). The Bonferroni correction was applied. The data and descriptive statistics for these measures were screened at the outset for potential errors and outliers. We discovered two outliers when checking for normality. One was found on the 'Safe and stimulating learning climate' scale at timepoint 3 and another was found on the 'Clarity of instruction' scale at timepoint 4. These datapoints were winsorized, but the resulting transformation did not impact the results. Therefore, the original data were used for data analyses.

Next, post hoc analyses were conducted to investigate the differences between teacher factors after checking sphericity using Mauchly's test. Finally, a paired sample t-test was conducted to compare the differences between T3 and T4 (before and after the intervention) and the differences between T1-T3 (baseline), controlled for multi-testing.

Results

Effect of teacher professional development program

Table 1 shows the descriptive statistics. To answer the first research question, regarding the effects, paired samples t-tests were run. We found that the professional development program had the following effects on dynamic math interviews (Table 1): during the post-test math interview, teachers asked significantly more questions about children's mathematics experiences, asked more questions about children's reasoning and problem-solving processes, created a safer and more stimulating climate and summarized their children's educational needs more often. The posttest dynamic math interviews focused on more aspects of children's mathematical development than the pretest interviews did.

The results showed that the teacher professional development program had a less effect on other qualitative aspects of dynamic math interviews. For example, teachers did not ask significantly more questions designed to identify a child's specific needs and were not more focused on supporting these needs. When teachers gave support, it was most often verbal.

Table 1 Means and Standard Deviations for Scores on Pretest and Posttest Dynamic Math Interview Teacher Professional Development Program (N = 23)

		M (*SD*) pre-test	*M* (*SD*) post-test	*t*	*p*
1.	Questions focused on child's math experiences, beliefs, emotions	1.91 (3.72)	20.83 (12.84)	-7.00	.001**
2.	Questions focused on child's thinking and solving processes	24.09 (12.84)	37.83 (16.97)	-3.41	.003*
3a.	Questions to identify child's needs in mathematics in general with actively involving student's voice	.04 (.21)	.52 (.85)	-2.71	.013
3b.	Questions to identify child's instructional needs	.04 (.21)	.39 (1.08)	-1.79	.088
3c.	Questions to identify child's needs regarding content and methods	.00 (.00)	.09 (.29)	-1.45	.162
4.	Questions to check child knows the right answer	3.61 (2.33)	3.43 (2.63)	.25	.807
5.	Questions to determine the level and adequacy of child's prior knowledge and understanding	.26 (.86)	.48 (.51)	-1.00	.328
6a.	Giving support by structuring	.48 (.59)	.70 (.47)	-1.31	.203
6b.	Giving support by reducing complexity	.04 (.21)	.13 (.34)	-1.00	.328
6c.	Giving verbal support	.87 (.34)	1.00 (.00)	-247	.022
6d.	Giving support by using representations of real situations	.13 (.34)	.17 (.39)	- .04	.665
6e.	Giving support by using models or schemes	.13 (.34)	.30 (.47)	-1.45	.162
6f.	Giving support by material support	.04 (.21)	.18 (.39)	-1.82	.083
6g.	Giving support by modelling	.00 (.00)	.09 (.29)	-1.45	.162
7.	Safe and stimulating climate during dynamic math interview	2.48 (.95)	3.48 (.51)	-5.30	.001**
8.	Teacher summarizes the math learning needs	.70 (1.89)	2.43 (1.24)	-3.83	.001**
9.	Dynamic math interview is focused on a wide scope	2.74 (.69)	1.35 (.78)	7.09	.001**

Note. $p^* \leq .01$ (after Bonferroni correction 0,05/17 = 0.0029), $p^{**} \leq .001$

4

Effect of the intervention on teacher factors of mathematics teaching

The second research question addressed the effect of the teacher professional development program and practice period of dynamic math interviewing on teachers' actual mathematics teaching behavior and their perceived math self-efficacy and mathematical knowledge for teaching. Descriptive statistics, means, and standard deviations for the different timepoints of measures, are presented in Table 2.

Table 2 Measures of Teacher Factors at Four Timepoints (N = 23). The Intervention was Between T3 and T4

	T1 *M* (*SD*)	T2 *M* (*SD*)	T3 *M* (*SD*)	T4 *M* (*SD*)
1. Actual mathematics teaching behavior	2.82 (0.29)	3.01 (0.42)	3.06 (0.37)	3.33 (0.36)
a. Safe and stimulating learning climate	3.50 (0.38)	3.56 (0.51)	3.78 (0.36)	3.79 (0.35)
b. Efficient classroom management	3.48 (0.41)	3.45 (0.45)	3.65 (0.51)	3.77 (0.29)
c. Clarity of instruction	2.97 (0.39)	3.23 (0.48)	3.42 (0.50)	3.62 (0.42)
d. Activating learning	2.74 (0.47)	3.07 (0.47)	3.04 (0.44)	3.40 (0.36)
e. Differentiation and adapting lesson	2.33 (0.75)	2.68 (0.85)	2.42 (0.78)	3.02 (0.50)
f. Teaching learning strategies	2.05 (0.53)	2.51 (0.49)	2.30 (0.48)	2.77 (0.46)
g. Math-specific teaching strategies	2.55 (0.64)	2.60 (0.51)	2.58 (0.65)	3.10 (0.54)
2.Mathematics teaching self-efficacy	7.12 (0.43)	7.20 (0.45)	7.15 (0.39)	7.47 (0.37)
3.Mathematical knowledge for teaching	3.16 (0.37)	3.27 (0.34)	3.19 (0.33)	3.43 (0.35)

To examine the effect of the intervention on teacher factors (research question 2), we first conducted repeated-measures ANOVA to compare differences between timepoints (baseline T1, T2, T3, and before and after the intervention T3-T4). Next, we computed post hoc tests to confirm where the differences occurred between T3 and T4. Finally, we computed a paired samples *t*-test to compare the differences between baseline and T3-T4. The results are displayed in Table 3.

Baseline T1, T2, T3:

The results showed an overall effect on two scales of mathematics teaching behavior across three timepoints of the baseline main scores. The scales were 'Clarity of instruction' and 'Teaching learning strategies'. There was no increase in the baseline mean scores of the other variables over time.

Table 3 Development of Teacher Factors (N = 23)

Teacher Factors	Baseline T1-T2-T3				T3-T4			
	λ	F	p	η_p^2	λ	F	p	η_p^2
1. Actual mathematics teaching behavior	.73	5.06 (2,42)	.011	.19	.62	13.60 (1,22)	.001**	.38
1a. Safe and stim. learning climate	.71	3.79 (2,42)	.031	.15	.99	0.02 (1,22)	.898	.00
1b. Efficient classroom management	.90	1.19 (2,42)	.315	.05	.94	1.41 (1,22)	.248	.06
1c. Clarity of instruction	.61	6.66 (2,42)	.001**	.24	.87	3.37 (1,22)	.080	.13
1d. Activating learning	.71	5.09 (2,42)	.010	.20	.52	20.58 (1,22)	.001**	.48
1e. Differentiation and adapting lesson	.86	1.80 (2,42)	.180	.08	.52	20.58 (1,22)	.001**	.48
1f. Teaching learning strategies	.64	6.49 (2,42)	.003*	.24	.54	18.99 (1,22)	.001**	.46
1g. Math-specific teaching strategies	.99	0.11 (2,42)	.900	.01	.48	23.91 (1,22)	.001**	.52
2. Mathematics teaching self-efficacy	.95	0.39 (2,42)	.677	.02	.66	11.26 (1,22)	.001**	.34
3. Mathematical knowledge for teaching	.87	1.11 (2,42)	.340	.05	.65	11.64 (1,22)	.001**	.35

Note: p * ≤ .005 (after Bonferroni correction 0,05/10 = .005), p** ≤ .001

4

T3 [-Intervention-] T4:

The results showed an overall effect (overall, all scales together) on mathematics teaching behavior over time, across two timepoints – T3 (before the intervention) and T4 (after the intervention) (Table 3:1). No overall effect over time was found on the scales ‘Safe and stimulating learning climate’, ‘Efficient classroom management’ and ‘Clarity of instruction’ (Table 3:1a, 1b, 1c). The effect over time on ‘Clarity of instruction’ was significant at the baseline, but not across T3 and T4. Results showed an overall effect over time on ‘Activating learning’, ‘Differentiation and adapting lesson’, ‘Teaching and learning strategies’ and ‘Math-specific teaching strategies’ (Table 3:1d, 1e, 1f, 1g). These scales represent more advanced levels of teaching behavior. The results showed an overall effect between T3 and T4 on teachers’ ‘Mathematics teaching self-efficacy’ (Table 3:2) and ‘Mathematical knowledge for teaching’ (Table 3:3).

To summarize, there were significant time-related effects between T3 and T4 --before and after the intervention-- related to three teacher factors: mathematics teaching behavior, teachers' sense of mathematics teaching self-efficacy and teachers' beliefs in mathematical knowledge for teaching.

We also conducted a paired samples *t*-test to compare the differences between T1-T3 (baseline), and T3 and T4 (before and after the intervention), corrected for multi-testing ($p < .005$). Improvement between T3 and T4 was greater than between T1-T3 on the following teacher variables: mathematics teaching behavior (overall) as well as the specific scales 'Activating learning', 'Differentiation and adapting lesson', 'Teaching and learning strategies' 'Math-specific teaching strategies', teachers' mathematics teaching self-efficacy and mathematical knowledge for teaching. These findings corroborated the ANOVA results.

These findings reveal that there was a significant increase in teacher factors – mathematics teaching behavior (overall score and the scales of teaching behavior at an advanced level), teachers' sense of mathematics teaching self-efficacy and teachers' beliefs in mathematical knowledge for teaching – between the start and the end of the intervention.

Discussion

The aim of this study was to examine the effect of a teacher-child dynamic math interview teacher professional development program on the quality of dynamic math interviews with fourth graders, the effect of dynamic math interviews on mathematics teaching behavior, teachers' sense of mathematics teaching self-efficacy and teachers' beliefs in mathematical knowledge for teaching within the elementary school context. The results showed that the teacher professional development program had a positive effect on the quality of dynamic math interviews. Furthermore, results showed an effect of the intervention on all teacher factors (teaching behavior, mathematics teaching self-efficacy, mathematical knowledge for teaching).

Effect of the teacher professional development program

The positive effect of the teacher professional development program is consistent with the findings of other research that links specific design characteristics to professional development influence (e.g., Desimone, 2009; Heck et al., 2019; Van Driel et al., 2012). In the present study, the training was focused on content related to dynamic math interviews and effective mathematics teaching. Examples of the active and practice-based learning methods used are good examples of math interviews, discussions about mathematics teaching articles and analysis of tasks and errors. Coherence was achieved by focusing on the policy standard goals of mathematics teaching in primary education, adjusting to the identified needs of teachers prior to the teacher professional development program and using the same supportive tool for dynamic math interviews. The teacher professional development program also achieved collective participation. In the present study, only fourth grade school teachers with a purposeful focus on the same subject, mathematics, participated in the teacher professional development program and collaborated during the meetings.

Furthermore, the use of videos as a core component supported teacher learning. This corresponds to other studies that used videos taken in teachers' own teaching practice as part of teacher professional development (e.g., Borko et al., 2011; Tripp & Rich, 2012). In their research Heck et al. (2019) emphasized the importance of using videos and expert facilitators able to scaffold teachers' viewing and discussion and promote open, thoughtful dialogue. This was also the case in the current study. Teachers gave each other feedback on the dynamic math interviews based on observation and discussion. This element of professional development, an active practice-based way of learning focused on diverse aspects of teacher-child interaction related to mathematics, appeared to be an effective feature of the teacher professional development program.

The novel and innovative features of this study included focus on the knowledge and skills required for dynamic math interviews and the development of a supportive tool for conducting these interviews. The tool incorporated features needed to conduct an interactive, solution-driven, future-focused dynamic math interview that actively involved the child in his or her own mathematical development (Allsopp et al., 2008; Bannink, 2010; Lee & Johnston-Wilder, 2013; Pellegrino et al., 2001). The tool was aimed at supporting teachers' diagnostic skills and mathematical knowledge for teaching (Hill et al., 2008; Hoth et al., 2016).

To summarize, the teacher professional development program in the present study was based on the aforementioned characteristics of effective professional development; this may have contributed to the positive effect of the program on the quality of dynamic math interviews.

Relationships between the intervention and teacher factors

As had been expected, findings show that the intervention had an effect on actual mathematics teaching behavior and perceived mathematics teaching self-efficacy and mathematical knowledge for teaching. The intervention where teachers conducted dynamic math interviews with fourth grade children to better understand their reasoning and understanding, preconceptions, misconceptions, strategies, math experiences, emotions, strengths and needs, was positively related to advanced aspects of mathematics teaching.

Firstly, the effects of the intervention on teaching behavior during mathematics lessons were seen on all scales of actual mathematics teaching ('Activating learning', 'Differentiation and adapting lesson', 'Teaching and learning strategies', and 'Math-specific teaching strategies'). The effects were significant on the more complex teaching behaviors 'Differentiation and adapting lesson' and 'Teaching and learning strategies' (e.g., Van der Lans et al., 2018). The effect was also significant on the supplemental scale -- 'Math-specific teaching strategies' – another complex teaching behavior. This supplemental observation instrument, specifically addressing mathematics teaching, was closely related to other aspects, such as the use of representations and attention to solving processes and metacognitive skills. Because of this, teaching behaviors, especially those at an advanced level, improved in this study. This was in line with the work of Porter et al. (2000). In their study, transfer of the teacher professional development program was most often seen in more complex teaching strategies when the program had high quality characteristics, such as active learning, consistency and coherence, as was the case in the present study. Other research (Deunk et al., 2018) has suggested that an informed view of both children' understanding of mathematics and their math learning needs, could contribute to adaptive mathematics teaching in the classroom. The present study found that dynamic math interviews, which were central in the intervention, are an effective way to become informed and may effect teaching behavior in elementary school classrooms, in which teachers have to meet the diverse children's math needs.

However, results revealed no effect of the intervention on the following scales related to less complex teaching behavior: 'Safe and stimulating learning climate', 'Efficient classroom management', and 'Clarity of instruction'. It is difficult to interpret why. Teachers may have shifted their focus to more advanced aspects of mathematics teaching as a result of insights and experiences acquired during dynamic math interviews.

Secondly, effects were also found on teachers' perceived mathematics teaching self-efficacy and mathematical knowledge for teaching. This was in line with our hypothesis. The information obtained during dynamic math interviews benefitted teacher factors regarding mathematics teaching. This parallels the results of another study (Carney et al., 2016) in which a teacher professional development program focused on children's thinking, problem-solving and content knowledge specific to mathematics lead to an increase in teachers' mathematics teaching self-efficacy and mathematical knowledge for teaching. Previous research had shown that teachers with high mathematics teaching self-efficacy and mathematical knowledge for teaching better prepare and adapt their mathematics lessons (Chang, 2015; Hill et al., 2008; Nurlu, 2015). In the present study, the increase of teachers' perceived mathematics teaching self-efficacy and mathematical knowledge for teaching resulting from the intervention appeared to positively affect to the professional development and practice period.

Therefore, the results of the present study support the notion that the interaction between actual mathematics teaching behavior and perceived mathematics teaching self-efficacy and mathematical knowledge for teaching may be related to teachers' mathematics teaching practice. This is consistent with Wilkins (2008) and Charalambous (2015), who suggest that perceptions and knowledge interact to influence mathematics teaching behavior.

Study strengths, limitations, and directions for future research

The strength of this study lies, in part, in its longitudinal design. It covered two school years, involved the same teacher participants throughout the duration and reached across a variety of primary educational contexts (22 elementary schools, of varying sizes and varying populations, spread across the country). Moreover, it explored the associations between a particular intervention focused on learning and practice of teacher-child dynamic math interviews and teacher factors within the context of elementary mathematics education. Participation of the same teachers throughout allowed us to control variables which might otherwise influence reliability. Because of the quasi-experimental study design involving the same teachers during two years, no control group of teachers could be involved. Furthermore, the last measurement was taken shortly after the intervention period. A follow-up study could explore whether or not the recorded results are sustainable.

The present study is a first attempt to uncover the potential of a dynamic math interview professional development program as well as its practical application. To more broadly apply the findings, additional replication studies involving more teachers and a control group of teachers are needed. Although a small teacher sample size is common in studies which use classroom observations and coded videos as part of the intervention, it may limit the use of the findings.

While we kept in touch with the heads of school throughout the study, emphasizing the need for them to support teacher participation, we did not take into account school contextual conditions such as the role of the school leader, demographics, or professional school culture (Opfer & Pedder, 2011). It would be useful to investigate if and how these influence teacher factors within the context of effective mathematics teaching in future studies.

Implications for practice

This study supports the notion that a teacher professional development program, based on effective characteristics of professional development in combination with a supportive scripted tool for dynamic math interviews, can influence the quality of teacher-child dynamic math interviews. The teacher professional development program design may be useful in other (research) contexts to improve mathematics teaching. The program might be improved by increasing opportunities for peer feedback concerning child support during a math interview. And the tool developed could help build a framework for dynamic math interviews.

Conducting dynamic math interviews adds value to mathematics teaching competencies. Interviewing children broadly on diverse aspects -- such as problem-solving processes, math experiences, math related beliefs, prior knowledge and skills -- that play a role in mathematical development and allow teachers actively involve children provides insight into the learning and educational needs of children. It also helps teachers meet these needs.

Conclusion

This quasi-experimental study is the first to explore the influence of dynamic math interviews on teachers' mathematics teaching behavior and perceived mathematics teaching self-efficacy and mathematical knowledge for teaching using an original research design involving same teacher participation over two years. Based on the promising results, we conclude that a teacher professional development program with effective characteristics contributed to improved teacher-child dynamic math interviews. In addition, conducting dynamic math interviews with children impacts mathematics teaching behavior, teachers' sense of mathematics teaching self-efficacy and teachers' beliefs in mathematical knowledge for teaching. Teachers seem to apply the lessons learned in dynamic math interviews in more complex and adaptive teaching behaviors and in their perceived mathematics teaching self-efficacy and mathematical knowledge for teaching. Dynamic math interviews seem to provide unique opportunities for teachers to identify and meet children's math learning needs. This study marks an important starting point in research on the effects of dynamic assessment as an approach for teachers to get an informed overview of children's mathematical development, their educational needs in learning mathematics and to be better able to adapt their mathematics teaching.

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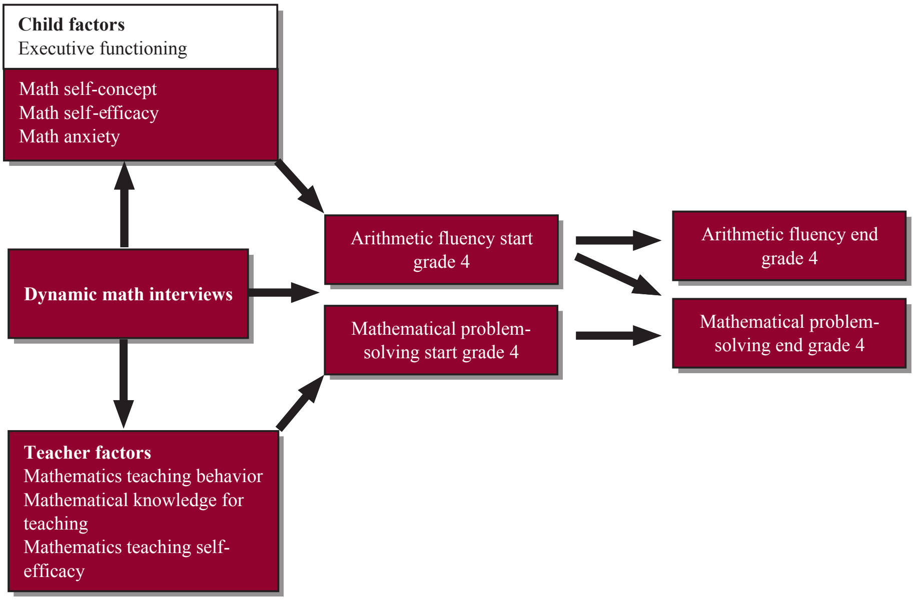
Chapter 5

Dynamic math interviews to identify children's math learning needs

A manuscript, based on this chapter, has been published:
Kaskens, J., Goei, S. L., Van Luit J. E. H., Verhoeven, L., & Segers, E. (2021). Dynamic maths interviews to identify educational needs of students showing low math achievement, *European Journal of Special Needs Education,* 1–15. https://doi.org/10.1080/08856257.2021.1889848

Abstract

We investigated the adequacy of the conduct and possible benefits of the use of dynamic math interviews with fourth grade children showing low mathematics achievement. This was done to facilitate the identification of their math learning needs and promote their subsequent mathematical development (mathematical problem-solving ability, arithmetic fluency, math self-efficacy, math self-concept, and math anxiety). In a pretest-posttest control group design, mathematical development was assessed at both the start and end of the school year. The experimental group had 19 fourth graders (children aged 8-10 years), showing low mathematics achievement and the control group had 15 such children. The intervention consisted of a dynamic math interview teacher professional development program and a practice period in which the teachers in the experimental group conducted an interview with each of the children. Qualitative analyses of the transcripts of the video-recorded interviews showed the conduct of the individual dynamic math interviews to be effective and to facilitate the identification and understanding of the math learning needs of children with low mathematics achievement. On the basis of such interviews, teachers provided feedback and support that were clearly attuned to the children's specific math learning needs. Children's mathematical development in the experimental and control groups did not differ significantly although differences in mathematical problem-solving ability were apparent. This study shows the potential of an analytical framework to evaluate the adequacy and benefits of dynamic math interviews in a more valid way by viewing relevant aspects in conjunction.



Introduction

The success of children's mathematics achievement accounts for considerable variance in educational outcomes but also impacts daily lives, self-reliance and later career opportunities. Persistent difficulties can occur in several domains of basic mathematics including learning arithmetic facts, retrieving these facts from long-term-memory, and the mastery and application of procedures for solving mathematical problems (e.g., Andersson, 2008; Fuchs et al., 2016; Geary, 2004, 2011; Mazzocco, 2007). Identifying and meeting the specific needs of children with low mathematics achievement is a major challenge for teachers in general and those with inclusive classrooms in particular (Mitchell, 2015). To successfully understand the math learning needs of low math achievers, teachers need insight into their mathematical performance, thinking, understanding, and beliefs (Deunk et al., 2018). However, current mathematics assessment is dominated by standardized, norm-referenced testing with its focus on the products of student learning as opposed to requisite math solving strategies, underlying thought processes, learning potential, and math-related beliefs and emotions (Allsopp et al., 2008). A promising alternative is the use of the dynamic math interview: a flexible, process-oriented, semi-structured assessment approach that can help identify the specific educational needs of children and particularly those with low mathematics achievement (Wright et al., 2006; Van Luit, 2019). In the present study, we implemented a dynamic math interview within the context of mathematics learning in elementary schools in order to better identify children's math learning needs in perspective of better mathematics achievement (Allsopp et al., 2008; Ginsburg, 1997, 2009).

The developmental courses of average and low math achievement

In the first years of elementary school, children are expected to develop an understanding of numbers, counting, and basic arithmetic skills or the prerequisites for later mathematical development (Geary, 2004). Starting in grade 4, the focus of mathematics education shifts to advanced mathematics (e.g., fractions, proportions) and more abstract mathematical problem-solving requiring more complex mathematics skills. In several longitudinal studies, strong associations have been demonstrated between early and later mathematics achievement (e.g., Watts et al., 2014). And in other research, the development of children's mathematics ability has been shown to be facilitated by the promotion of arithmetic fluency, an understanding of underlying concepts but also calculation principles and the formulation of solution plans for mathematical problem-solving (Andersson, 2008). Not only the cognitive aspects of mathematics learning but also the beliefs and emotions of children have been shown to impact their mathematical development (Chinn, 2012). Three aspects of beliefs and emotions have been shown in particular to be associated with mathematics achievement: self-efficacy, self-concept, and math anxiety (Lee, 2009).

Self-efficacy is a judgment of one's capacity to perform in general and domain-specific tasks in particular (Bandura, 1997). *Self-concept* is beliefs about one's competence and general self-worth but also — for example — one's math competence. Self-concept is, thus, more general than self-efficacy (Bong & Clark, 1999). A child, for instance, may have a generally positive math self-concept but hold very different self-efficacy beliefs with regard to specific math tasks. In a recent study of 600 fourth grade children, math self-concept was shown to be a significant positive predictor of the development of arithmetic fluency (Kaskens et al., 2020). *Math anxiety* is a negative emotional response to numbers and/or math-related situations (Suárez-Pellicioni et al., 2016). It has been shown to negatively correlate with mathematics achievement in general and complex, verbal, mathematical problem-solving in particular (Wa et al., 2017). And this negative correlation between math anxiety and mathematics achievement has been shown, in turn, to be due to: avoidance of mathematics, suppression of cognitive processing and other social factors (Maloney & Beilock, 2012). In general, better mathematics achievement correlates positively with self-efficacy and self-concept while lower mathematics achievement correlates negatively with math anxiety. By grade 4, moreover, the associations have been found to be reciprocal: self-concept influences mathematics achievement and vice versa (Weidinger et al., 2018).

Children showing low mathematics achievement are known to experience difficulties with both the basic and more abstract aspects of mathematical development (Fuchs et al. 2016; Träff et al. 2020). They have also been found to be more influenced by affective math-related factors than average math achievers (Lebens et al., 2011). All of this shows that not only cognitive factors but also the beliefs and emotions of children should be taken into account when identifying children's math learning needs.

Dynamic math assessment

Dynamic math assessment differs from traditional standardized testing on a number of fronts. First, dynamic testing procedures all have an intervention or training phase for students, which is aimed at the identification of how individual instruction can lead to improved achievement (Elliott et al., 2010; Fuchs et al., 2008). Second, in an interactive teacher-child dialogue, children demonstrate their mathematical understanding and thinking and mathematical knowledge/skill and teachers address specific errors, provide support and gain in-depth insight into the strengths and weaknesses of children (Ginsburg 1997, 2009; Pellegrino et al., 2001).

Dynamic math assessment could typically be conducted as a semi-structured interview in which the teacher undertakes process-oriented research to determine not only achievement levels but also the application and use of critical procedures and strategies to identify educational needs and suitable forms of instruction and (additional) support (Van Luit, 2019; Wright et al., 2006).

Dynamic math interviews have been shown to be an effective form of dynamic math assessment (Allsopp et al., 2008; Caffrey et al., 2008; Van Luit, 2019). Outcomes of dynamic math interviews are assumed to be informative in guiding classroom instructions and interventions. Explicit modelling, increased use of visual representations and/or manipulatives can be offered (e.g., use of imitation money, fraction circles) (Emerson & Babtie, 2014; Gersten et al., 2009). To our knowledge, only some existing scripted assessment tools are directed on a specific mathematics domain (e.g., Wright et al., 2006).

To date, the empirical evidence on the adequacy and actual benefits of dynamic math assessment is limited. In a review of four earlier studies (Caffrey et al., 2008), dynamic assessment was found to contribute unique variance to the prediction of future mathematics achievement and thus go *beyond* traditional static math assessment. In a study by Seethaler et al. (2012) involving the presentation of scaffolded mathematics content to first graders, a dynamic assessment approach was found to provide greater insight into the learning capabilities of the child relative to traditional assessment and particularly with regard to the child's word problem-solving.

In the past, Ginsburg (1997) suggested video-recording dynamic math interviews for subsequent review and discussion, the creation of guidelines for evaluation purposes, and the explicit assessment of inter-interpreter reliability. Further information on the validity and benefits of dynamic math assessment of educational needs is not available. Therefore, insight into the conditions needed to determine the validity of dynamic math interviews and the adequacy and the benefits of such an approach to identify math learning needs, is thus needed.

The ability of teachers to conduct dynamic math interviews

Dynamic math interviewing requires specific competencies, such as the ability to explore and expand the limits of child's knowledge and to interpret child's thinking (Ginsburg, 2009). The teacher must be able to stimulate child's responding and thereby gain a better understanding of a child's perspective (Empson & Jacobs, 2008; Lee & Johnston-Wilder, 2013). The interaction with children may often have a solution-focused character. Teachers then pose questions to help children identify their learning strengths and weaknesses but also questions aimed at stimulating children to share their mathematics learning experiences and emotions and specify their goals and the support needed to achieve these goals (Bannink, 2010). In order to become competent math interviewers, teachers must practice with the observation, posing appropriate questions, and adequate responding. Video recording of dynamic math interviews, training, reflection, and ongoing review purposes is critical (Wright et al., 2006).

In order to meet the educational needs of each and every child, teachers must recognize the diversity of learning trajectories and have the capacity to provide scaffolded support along the way (Deunk et al., 2018; Empson & Jacobs, 2008). Van de Grift (2007) identified the provision of a safe but stimulating learning climate, efficient classroom management, and clear instruction as necessary for effective teaching. Aspects of teaching such as showing children how to *simplify* complex problems have also been identified as critical aspects of effective mathematics teaching (Van der Lans et al., 2018). Teachers must have the required knowledge base but also knowledge of alternatives for stimulating children's mathematics learning (see Hill et al., 2008). Only then can teachers decide which alternative is most suited for a given child, in a given domain of learning, and a given problem at a given point in time. Thus, when teachers are better able to identify the math learning needs of children showing low mathematics achievement, they should be able to better establish meaningful instructional goals and make the necessary adaptations to their mathematics education (Hoth et al., 2016).

To support low math achievers, several studies have shown the following teaching strategies to be successful: highly structured and organised programmes; the giving of hints for problem solution; explicit modelling; use of visual representations and manipulatives; careful selection and sequencing of instructional examples; and having children verbalise their strategies but also those modelled by the teacher (e.g., Gersten et al., 2009).

The present study

Whether or not the dynamic math interview is an effective tool for identifying the math learning needs of children showing low mathematics achievement has yet to be demonstrated. We therefore posed the following two questions. 1) What is the adequacy of teachers' use of a dynamic math interview to identify the math learning needs of children with low mathematics achievement?; 2) To what extent does the use of a dynamic math interview promote the mathematics learning of children with initially low mathematics achievement?

Critical elements for the determination of the reliability, validity, and benefits of using a dynamic math interview were identified and thus elements for the development of an analytic framework.

In order to help the teachers with the conduct of the dynamic math interviews, a scripted protocol was developed on the basis of the learning assessment model of Pellegrino et al. (2001), the interview model of Delfos (2001), and the available research on dynamic educational assessment (e.g., Allsopp et al., 2008; Bannink, 2010; Black et al., 2004; Ginsburg, 1997, 2009).

We expected the conduct of dynamic math interviews to indeed help teachers identify the math learning needs of low math achievers. In addition, we expected that teachers demonstrating high levels of competence for the conduct of dynamic math interviews also show relatively better mathematics teaching behavior. The underlying assumption is that such teachers will benefit most from the use of dynamic math interviews to identify specific math learning needs of children, subsequently put this information into daily teaching practice, and thus promote the mathematical development of all children and those initially showing low mathematics achievement in particular. Observations of mathematics teaching behavior afforded us information on the levels of effective mathematics teaching behavior.

Method

Study design and participant selection

Data on children's mathematical development and teachers' actual mathematics teaching behavior were collected at the starts (T1) and ends (T2) of two consecutive school year. Year 1 constituted a control condition. Year 2, in which a dynamic math interview teacher professional development program was conducted followed by a practice period, constituted the experimental condition (see Figure 1).

Year 1: control group					
Aug-Sep	Mathematics taught as usual				June
T1					T2
Year 2: experimental group					
Aug-Sep	Oct	Nov-mid Feb	Feb	March-mid June	June
T1	*Individual feedback on a conducted DMI*	Teacher professional development program	*Individual feedback on a conducted DMI*	Practice period Individual data collection for each teacher with one child showing low mathematics achievement	T2

Figure 1 The Research Design
Note: DMI = Dynamic Math Interview.

Participants were recruited by open invitations via social media (Twitter) and direct mail (school principals and fourth grade teachers). An information meeting was held for interested teachers in two regions of the Netherlands and 23 teachers from 22 different schools agreed to participate in the end. Nineteen of these teachers, who conducted a dynamic math interview with a child showing low mathematics achievement, were involved in this study. The teachers were given printed information about the study and a factsheet about the data collection methods.

The 23 participating teachers were asked to identify children showing low mathematics achievement (i.e., scores below the 20th percentile on a criterion-based standardized Dutch national test) (Cito; Janssen et al., 2005). This was done for years 1 and 2 (different classes, same teachers). The mean score on this mathematics test for the entire group of children being taught by the 23 participating teachers in the *control* group (N = 591) was 217.61 (*SD* = 26.08) (range of 131-321) with 97 showing low mathematics achievement (scores < 193). The mean for the year 2 *experimental* group (N = 449) was 216.43 (*SD* = 28.19) (range of 110-312) with 92 children showing low mathematics achievement.

All of the 23 teachers participated in the teacher professional development program. Only 19 of the 23 teachers had children with low mathematics achievement in their classes, however, and therefore participated in the present study: 3 men and 16 women with an average of 11.6 years of teaching experience (*SD* = 9.63, range 3-40). Thirteen had a Bachelor's degree in education (68%), five had additional graduate training (26%), and one had a Master's degree.

Each of the 19 participating teachers conducted a dynamic math interview with a child with a mathematics score below the 20th percentile criterion on the Cito test. The dynamic math interview was conducted during the practice period in year 2 and video-recorded for data collection purposes. These children along with their teachers who performed the dynamic math interviews, constituted the *experimental* group (*n* =19). The mean age of the children was 9.26 years (*SD* = 0.41): 12 boys, 7 girls. To form a *control* group, peers in year 1 (i.e., prior to the dynamic math interview intervention) but taught by the same teachers as for year 2 and showing low mathematics achievement were sought. Only 15 children could be identified in such a manner; their mean age was 9.39 (*SD* = 0.47) (4 boys, 11 girls).

The sample was treated in accordance with institutional guidelines as well as APA ethical standards. Schools, parents, and children were informed about the purpose of the research, duration of the study, and procedures. Both teachers and parents provided active informed participation consent.

Measurement instruments

Mathematics teaching behavior

The International Comparative Analysis of Learning and Teaching (ICALT; Van de Grift, 2007; Van der Lans et al,. 2018) was used to observe 32 aspects of actual teaching behavior during mathematics lessons (7 scales). The first six observational scales address less complex to more complex teaching behaviors: providing a safe and stimulating learning climate; efficient classroom management; clarity of instruction; activating learning; teaching of learning strategies, and differentiation and adaptation of lesson. The seventh scale assesses children's involvement. Given that the ICALT is not math-specific, a supplemental eighth scale (S) for mathematics teaching strategies in particular was created (see Appendix A).

The eight items for the math-specific scale were developed by the first author in consultation with the co-authors for purposes of the present study. The items were based upon the levels of action as identified by Gal'perin (1978) and the model of problem-solving model of Polya (1957): 1) informal manipulation, 2) depiction of concrete mathematical actions and situations, 3) depiction of abstract models and diagrams, and 4) formal/abstract mathematical operations, 5) understand the problem and making connections between the previous four levels and using these connections to support lesson goals, 6) devise a plan, 7) carry out the plan/problem-solving process, and 8) check and interpret (see Appendix A). The internal consistency of the 8 scales considered together was good ($\alpha \geq 0.89$). This was also the case for the individual scales ($\alpha \geq 0.85$). The scoring for each of the 40 observational items was done along a 4-point Likert scale ranging from 1 (= *predominantly weak*) to 4 (= *predominantly strong*) and conducted by two independent mathematics teaching experts (the first author and a second observer, who were both trained and certified to use the ICALT). The inter-rater reliability was found to be good ($\alpha = 0.86$).

Children's mathematics achievement, beliefs, and emotions

Mathematical problem-solving. The first measure of mathematics achievement consisted of the criterion-based standardized Dutch national tests commonly administered at the middle and end of each school year to monitor student progress (Cito; Janssen et al., 2005). The tests present mathematical problems in variety of manners from several domains: using only mathematical notation or various combinations of text, pictures, and mathematical notation. The internal reliability in the present study was good (year 1 $\alpha = 0.87$; year 2 $\alpha = 0.81$).

Arithmetic fluency. The second measure of mathematics achievement was the Speeded Arithmetic Test (TTA; De Vos 2010), a standardized paper-and-pencil test containing four categories of math with 50 fact problems each: addition (difficulty varying from 6 + 0 to 29 + 28), subtraction (varying from 4 – 2 to 84 – 38), multiplication (varying from 4 x 1 to 7 x 9), and division (varying from 6 : 2 to 72 : 9). Children are given two minutes per math category. A correct answer yields one point for a total of 50 possible points per category of math and 200 for the total test. In the present study, the reliability and validity of the scales was good (total α = 0.86; addition α = 0.82; subtraction α = 0.80; multiplication α = 0.76; division α = 0.76).

Math self-concept, math self-efficacy, and math anxiety. The Mathematics Motivation Questionnaire for Children was used to measure math-related beliefs and emotions (MMQC; Prast et al. 2012). The questionnaire consists of five scales: math self-efficacy (6 items), math self-concept (6 items), math anxiety (5 items), math task value (7 items), and math lack of challenge (6 items). All items are responded to along a four-point scale: 1 = NO! (strongly disagree), 2 = no (disagree), 3 = yes (agree), 4 = YES! (strongly agree). Of particular interest for the present study with low math achievers were the self-efficacy, self-concept, and math anxiety scales. The internal consistency for two the scales in our study was acceptable: math anxiety (year 1, α = 0.79; year 2 α = 0.77) and math self-efficacy (year 1 α = 0.79; year 2 α = 0.77). The internal consistency for math self-concept was good (year 1 α = 0.85; year 2 α = 0.86).

Analytic framework

Using the method of qualitative content analysis as developed by Mayring (2015), we developed an analytic framework to examine the video-recorded dynamic math interviews (see Figure 2). The framework encompassed aspects of dynamic assessment considered critical for a dynamic math interview to be effective. For purposes of the present study, we focused on 10 aspects judged to be critical for the identification of children's math learning needs and thus providing a stepping stone for meeting the needs. Three validation sessions were conducted in which five mathematics teaching experts (one validation session), eight researchers (one validation session) and a mathematics teaching expert with a university master's degree in special education (one validation session) coded transcripts with concepts from the tentative analytic framework. Following each validation session, the analytic framework and accompanying manual were adjusted and refined. Several codes, for example, were added to identify the types of questions posed by the teachers and the type of support provided. Directions for the coding of the questions posed by the teachers were made more specific and refined. We also added coding of the adequacy of teacher responding to children to the analytic framework.

The first author coded all of the transcribed dynamic math interviews. An additional mathematics teaching expert with a Master's degree in special education but blind to the aims and design of the present study coded a random selection of six transcripts using the analytic framework. The inter-rater reliability for the scoring of the six transcripts was found to be good with a consensus norm of 81% agreement.

1. *Ratio open to closed questions posed by teacher.* Open questions are assumed to elicit greater information and therefore preferred over closed questions. At the start of the dynamic math interview, closed questions may nevertheless be more suitable for the purpose to establish trust or to check the teacher has understood the child correctly. By asking in-depth questions, the teacher can gain more information or clarity. The proportion open questions should be higher than the proportion closed questions.
2. *Questions focused on child's math experiences, beliefs, and emotions.* With the intention of a wide scope for the dynamic math interview, the teacher can also ask questions addressing child's math experiences, beliefs, and emotions. The percentage of the total number of posed questions focused on this aspect should be more than 20% of all questions of the dynamic math interview to be judged adequate.
3. *Questions focused on child's thinking and problem-solving processes*. These questions help gain insight into what the child understands and does not understand. The teacher can obtain an explanation for why the child does not understand things or cannot complete the problem correctly. The percentage of the total number of questions posed is calculated and should be higher than the percentage product-directed questions (aspect 4).
4. *Questions to check the child knows the right answer.* With these questions the teacher can gain information about mathematics achievement levels and mastery of skills. The attainment of process information as opposed to product (i.e., outcome) information should nevertheless prevail for the dynamic math interview to have added value near standardized tests. The percentage of the total number of questions posed is counted.

5. *Questions to identify math learning needs by actively eliciting 'student's voice'.* By posing questions with a solution-focused character the teacher can help the child begin moving towards solutions and future regarding mathematics learning. Have you ever had great math help? What did the person who gave you that do? What do you need to reach your next math learning goal? are examples of questions that elicit student's voice. Also increasing waiting time after posing a question can maximise the chances of gaining insight into the child's own thinking, the child's ideas, the promotion of commitment, and increased ownership. The percentage of the total number of questions posed is counted and should be at least 10% for the dynamic math interview to be judged adequate.
6. *Support given.* The teacher can provide support during a dynamic math interview. We distinguished: a) stimulating the child to write down steps in thinking, b) verbal support (e.g., hints), c) verbal support provided by notes by the teacher, d) material support (e.g., manipulate with imitation money), e) use of concrete representations of abstract models, f) use of representations of concrete mathematical actions and situations, g) clear structuring of problem/task, h) reduction of complexity, i) demonstration, and j) modelling. Support provided 4 times or more is indicated as most frequently provided support. Most important is that the support be appropriate.
7. *Adequate responding.* When a teacher responds to what a child says or does, they must do this in a manner which allows the child to take advantage of their response. This requires extensive mathematical knowledge. Adequate responding requires: insight into possible misunderstandings, provision of not only clear but also complete support, correct interpretation of child's mathematical statements, determination of appropriate support, and effective timing of the support. On the basis of this information, adequacy of responding can be assigned a score between 1 (= to a very small extent) and 4 (= to a very large extent), with a score ≥ 3 indicating adequacy.
8. *Creation of safe and stimulating climate.* Particularly for the conduct of a productive dynamic math interview, several conditions must be met: creation of a sufficiently warm and relaxed atmosphere, showing of respect, starting with a mathematical problem on which the child is likely to succeed, encouraging verbalisations, sincerity, and supportive remarks. This aspect of the dynamic math interview is assigned a score between 1 (= to a very small extent) and 4 (= to a very large extent), with a score ≥ 3 indicating adequacy.
9. *Teacher summary of educational needs*. When the teacher succinctly reproduces what lies at the core of the child's needs, using the child's own words, this shows that the teacher has been listening carefully. It also allows the teacher to check their understanding of the child's math learning needs and goals. Co-responsibility on the parts of the teacher and child is also fostered. Summary of math learning needs assigned a score of 0 (= not) or between 1 (= to a very small extent) and 4 (= to a very large extent), with a score ≥ 3 indicating adequacy.
10. *Scope of the dynamic math interview.* A beneficial dynamic math interview must address various aspects of a child's mathematical development: thinking and problem-solving abilities; math-related experiences, beliefs, and emotions; and active involvement in the identification what they need for successful mathematics achievement. We distinguished five types of dynamic math interview scope, with the widest (a) being most preferred: a) teacher focus on child's mathematical thinking and problem-solving; math experiences, beliefs, and emotions; and active involvement in identification of needs; b) teacher focus on mathematics achievement; math experiences, beliefs, and emotions; c) teacher focus on math experiences, beliefs, and emotions; active involvement in identification of needs; d) teacher focus on mathematics achievement; active involvement in identification of needs; and e) focus solely on mathematics achievement.

Figure 2 Analytic Framework

Procedure

After participants were recruited, an information meeting was organised in two regions of the Netherlands. The teachers were given printed information about the study and a factsheet about the data collection methods. Later via e-mail, the teachers consented to being observed and video-recorded during the teaching of a regular mathematics lesson on the topic of fractions or ratios. Each teacher was observed and recorded teaching a mathematics lesson on two occasions (T1, T2). The lessons were scored using the ICALT+S. And the teachers were debriefed following observation.

Paper and pencils versions of the MMQC and TTA were administered in the classroom. Administration lasted approximately 35 minutes. The Cito mathematics achievement data were obtained from the participating teachers, with parental consent. On the same day, the teacher taught a mathematics lesson, which was recorded and scored using the ICALT+S. The teachers were debriefed after measurement and later informed of the results.

The *intervention* entailed a teacher professional development program consisting of four meetings with a duration four hours each, followed by a period of dynamic math interview practice. The program followed the design features recommended for professional development training purposes (e.g., Van Driel et al., 2012). The professional development program prototype was reviewed by experts and fine-tuned several times. The first author, an expert teacher trainer, organised and conducted the sessions. The program included an explanation of the protocol for a dynamic math interview, mathematical knowledge for teaching related to dynamic math interviews (e.g., understanding student errors), video examples of dynamic math interviews, and peer feedback on practiced and video recorded dynamic math interviews. Each teacher also received individual feedback from the teacher trainer on two occasions: once before the first meeting and once after the last meeting. On these two occasions, the teachers were asked to conduct dynamic math interviews for three self-selected word math problems from the Cito mathematics test in a manner they considered suitable. During the subsequent dynamic math interview practice period, the 19 teachers conducted and recorded the dynamic math interviews with the 19 children participating in the study. These videos, which varied in length, were fully transcribed and coded.

Data analyses

To answer the first research question, we initially analyzed the 19 videos qualitatively and then quantified the data.

To answer the second research question, a Wilcoxon signed-rank test was computed. We first checked the data for normality. Skewness-kurtosis were all within acceptable range (-1,1 and -2,2), but the Shapiro-Wilk test of normality showed only normal distributions for mathematical problem-solving, category addition of arithmetic fluency, and math self-efficacy. Due to the small sample size and non-normal distributions that were found, we computed the non-parametric Wilcoxon signed-rank test to compare the mathematical development of the control and experimental groups and the different groups over time. *P*-values are sensitive to sample size. Therefore, we considered the *p*-value in combination with calculation of effect sizes using Hedge's *g.,* a measure of effect size when sample sizes are different (n=19; n=15).

Results

Addressing the first research question, Table 1 presents the results of the qualitative analyses of the 19 dynamic math interviews in terms of adequacy of the dynamic math interviews (10 coded aspects; see Figure 2). The data on the summary score of changes in mathematics teaching behavior from T1 to T2 (i.e., before and after participation in teacher professional development program) and the children's individual mathematical development are also presented in Table 1.

A short summary of the 10 aspects:

1: Ratio of open to closed questions used by teacher.

2-5: Proportion of total number of questions with focus on: 2) child's math experiences, beliefs, and emotions; 3) child's thinking and problem-solving processes; 4) checking that the child knows correct answer; or 5) identification of child's math learning needs by actively eliciting student's voice.

6: Most frequently provided support (i.e., four or more times during dynamic math interview): a) stimulating the child to write down steps in thinking, b) verbal support, c) verbal support provided by notes by the teacher, d) material support e) use of concrete representations of abstract models, f) use of representations of concrete mathematical actions and situations, g) clear structuring, h) reduction of complexity, i) demonstration, and j) modelling.

7-8: Adequacy of responding (7) and providing a safe and stimulating climate (8). These aspects of the interviews were scored along a scale of 1 (= to a very small extent) to 4 (= to a very large extent).

9: Teacher summary of child's math learning needs was scored along a scale of 0 (= not) to 1 (= to a very small extent) to 4 (= to a very large extent).

10: Scope of the dynamic math interview was distinguished using five categories of responding (a-e): teachers who focuses on the child's math thinking and problem-solving; the child's mathematics experiences, beliefs, and emotions; and actively involving the child in the identification of their math learning needs showing the most wide scope (a).

Adequacy

Our analysis of the dynamic math interviews provided an abundance of information. Only the highlights of the findings of relevance to our research question presented in Table 1 are outlined here. In Figure 3 some examples of the analyses of the data summarized in Table 1 are described in more detail.

All of the 19 teachers were found to ask more open than closed questions in the analyzed dynamic math interviews. For 14 of the teachers (73.7%), more than 20% of their questions addressed the math experiences, beliefs, and emotions of the child. Sixteen (84.2%) asked more process- than product-oriented questions (i.e., focused on children's math thinking and problem-solving). Twelve of the dynamic math interviews (63.2 %) showed a wide range of attention and thus addressed: children's math thinking and problem-solving; children's mathematics experiences, beliefs, and emotions; and active involvement of children in the identification of their mathematical learning needs. Fourteen teachers (73.7 %) showed adequate responding (≥ 3), 16 teachers (84.2 %) created an adequate safe and stimulating climate (≥ 3). Eight teachers (42.1 %) summarized mathematical learning needs to an adequate extent (≥ 3). The most frequently provided support was verbal support: 17 teachers (89.5%) provided verbal support more than four times during the dynamic math interview.

With regard to the range of teacher performance in the dynamic math interviews, six teachers (31.6%) showed a high degree of attention to child's math thinking and problem-solving, on the one hand, and active involvement of children in the identification of their math learning needs, on the other hand (> 20% of all questions). The latter is also reflected in the extent of identified and explicitly verbalized math learning needs: a larger number of needs (range 6-11) was cited in the dynamic math interviews of teachers *5, 8, 10, 11, 12, and 15.* In the other dynamic math interviews, teacher *3* mentioned only one child need; *16* two needs; and *19* no needs. See also Figure 3 and some examples of dynamic math interviews in Appendix D.

The qualitative analyses and criteria described in Figure 2 show adequate dynamic math interviews for teachers *2*, *5*, *8*, *10, 11,* and *12*. A good balance was found in the types of questions posed (aspects 1-5); a wide range of topics was addressed (aspect 10); and adequate support and responding was given (aspects 6 and 7). A safe and stimulating learning climate was created (aspect 8). A summary of the child's math learning needs was supplied (aspect 9). In these dynamic math interviews, various aspects of a child's mathematical development were addressed by adequate teacher-child interaction with the aim to identify child's math learning needs. Positive associations were found for all aspects of child 2 *(*C*2)* mathematical development, for all aspects except reduction of math anxiety (C*5*), all aspects except self-efficacy and math anxiety (C*8*), all except self-efficacy and self-concept (C*10* and C*11*), all except self-efficacy, self-concept, and math anxiety (C*12*).

Four the aforementioned teachers (*2, 5, 11, 12*) showed high scores for *actual mathematics teaching behavior* (> 3) on both measurement occasions and two (*8, 10*) showed increases on the second occasion (T1 < 3, T2 > 3). The one dynamic math interview that was judged to be less than adequate was conducted by teacher *16*. It showed an insufficient balance between the different types of questions (aspects 1-5); a small scope (aspect 10); inadequate support and responding (aspects and 7); little or no creation of a safe and stimulating learning climate (aspect 8) and no summary of math learning needs (aspect 9). For this teacher, low actual mathematics teaching behavior scores were also found on both occasions (T1 and T2 < 3). It should nevertheless be noted that not all teachers showing high teaching behavior scores (T1, T2 >3) conducted dynamic math interviews which were judged to adequate on all aspects (teachers *4, 13, 14, 15, 19).* Conversely, not all teachers showing low teaching behavior scores (T1, T2 < 3) conducted dynamic math interviews which were judged to be inadequate on all aspects (teachers *7, 9, 16, 17,18*). All teachers have their strengths and weaknesses.

Table 1 Aspects of Dynamic Math Interviews in Relation to Teachers' Mathematics Teaching Behavior and Children's Mathematical Development

T	Mathematics-teaching behavior		1.	2.	3.	4.	5.	6.	7.	8.
	T1	T2	% open	%	%	%	%	≥4 times	1-4	1-4
1	2.96	3.44	54.84	19.36	47.31	10.75	8.06	b	3	3
2	3.47	3.67	54.00	42.00	38.00	8.00	10.00	a,b	4	4
3	2.86	3.40	59.02	14.06	31.15	36.07	1.64	b	3	4
4	3.32	3.68	67.44	2.33	48.84	20.93	11.63	b,e	4	4
5	3.52	3.68	87.50	21.88	31.25	18.75	26.56	-	4	4
6	3.31	2.95	56.76	25.23	33.33	22.52	17.12	a,b,c	2	3
7	2.46	2.97	66.07	23.21	35.71	23.21	3.57	b	1	2
8	2.91	3.31	72.09	21.28	21.11	11.70	37.87	a,b	3	4
9	2.85	2.97	64.29	20.00	35.71	22.86	8.57	b	3	4
10	2.16	3.38	80.65	40.32	12.90	16.13	22.58	b	3	4
11	3.15	3.48	71.74	28.28	32.61	14.49	21.74	a,b	4	4
12	3.57	3.70	75.38	14.29	25.00	9.52	34.52	a,b,e	4	4
13	3.63	3.75	54.70	54.70	15.39	8.55	10.26	b	2	2
14	3.44	3.30	87.32	32.39	29.58	26.76	5.63	b	3	3
15	3.33	3.70	50.00	50.00	20.00	0.00	33.33	b	4	4
16	2.90	2.83	75.00	6.25	45.83	33.30	0.00	b	1	2
17	2.81	2.73	60.20	17.20	23.66	21.51	9.68	a,b,c	3	4
18	2.96	2.68	71.90	46.15	7.69	15.39	10.26	d	3	4
19	3.22	3.54	57.45	25.93	51.06	8.51	0.00	b,c	2	3

Note: T = Teachers, C = Children ; 1: Ratio of open to closed questions used by teacher; 2-5: Proportion of total number of questions with focus on: 2) child's math experiences, beliefs, and emotions; 3) child's thinking and problem-solving processes; 4) checking that child knows correct answer; or 5) identification of child's math needs by actively eliciting child's voice; 6: Most frequently provided support; 7: Adequacy of responding; 8: Providing a safe and stimulating climate; 9: Teacher summary of child's educational needs; 10: Scope of the dynamic math interview.

Dynamic math interview 2 stands out in a positive way. This teacher showed a good level of actual mathematics teaching behavior to start with (> 3) with increased scores from T1 to T2. More open than closed questions were asked. There was variation across questions concerned with child's math experiences, beliefs, and emotions (42%); questions focused on child's mathematical thinking and problem-solving processes (38%); questions used to check that the child knows the right answer (8%); and questions showing the teacher to involve the child, give the child a voice (10%). This teacher clearly provided support during the dynamic math interview (four times by stimulating the child to write down the steps in thinking, six times by giving verbal support, and two times by clearly structuring the task). Responsiveness, Climate, and Summarizing child's needs also received high ratings, and the interview was judged to have a wide scope (a). The child's mathematical development increased from 187 (T1) to 216 (T2) in the experimental condition for math problem-solving and from 69 (T1) to 134 (T2) for arithmetic fluency. Furthermore, the child's math self-concept and self-efficacy clearly increased and their math anxiety clearly decreased from 27 (T1) to 12 (T2). Among the identified math learning needs were the following: step by step

9.	10.	C	Math. problem-solving		Arithmetic fluency (total)		Math self-efficacy		Math self-concept		Math anxiety	
0-4	a-e		T1	T2	T1	T2	T1	T2	T1	T2	T1	T2
2	a	1	140	202	76	86	13	11	12	9	15	22
4	a	2	187	216	69	134	7	17	13	16	27	12
1	b	3	190	208	64	68	12	14	18	14	10	7
3	d	4	186	198	91	147	20	20	24	23	10	13
4	a	5	191	225	65	97	17	18	13	18	12	12
4	a	6	186	230	67	88	18	17	21	22	7	7
2	b	7	190	223	130	142	17	18	23	20	9	6
4	a	8	154	238	65	82	18	16	8	13	17	19
2	a	9	189	237	112	146	-	19	-	18	-	10
4	a	10	188	250	68	108	9	9	10	9	22	20
4	a	11	187	206	41	66	17	16	18	17	15	9
4	a	12	189	242	90	120	19	18	20	19	10	11
1	a	13	170	214	80	86	17	19	19	19	11	10
2	b	14	179	197	17	30	12	12	11	10	13	14
2	a	15	191	258	82	95	15	13	13	9	19	15
0	e	16	193	181	79	53	17	20	19	20	9	8
1	a	17	188	207	69	96	12	15	13	15	13	13
1	b	18	188	226	112	80	18	18	19	20	10	16
0	b	19	183	229	68	104	17	19	12	16	18	9

instruction, writing down each small step during the solution process, writing down interim results, and checking of answers. This child requested a copy of the exercise sheets so that he could write directly on it during daily mathematics lessons.

Dynamic math interview 16 stands out in a negative way. This teacher demonstrated a low level of actual mathematics teaching behavior (< 3). While the teacher asked more open than closed questions, they asked relatively few questions about the child's math experiences, beliefs and emotions (6.3%) and did not involve the child to any extent. The teacher asked questions about the child's mathematical thinking and problem-solving solving processes (45.8%) but was also quite product-oriented (33.3%). Mostly verbal support was provided. Responsiveness, Climate, and Summarizing the needs of the child were rated as low, and the dynamic math interview was judged to have a restricted scope (e). In fact, the child's mathematical development showed a decrease from 193 (T1) to 181 (T2) for mathematical problem-solving ability and a decrease from 79 (T1) to 53 (T2) for arithmetic fluency. The child's math self-efficacy nevertheless increased three points. The math learning needs identified for this child were: read the mathematical problem thoroughly and repeatedly and pay attention to the word 'approximately'.

Dynamic math interview 19 shows a mixed picture. This teacher demonstrated a high level of actual mathematics teaching behavior (T1: 3.22, T2: 3.54). More open than closed questions were asked. A variety of questions was asked about the child's math experiences, beliefs, and emotions (25.9%) with a predominance of process-oriented (51.1%) over product-oriented (8.5%) questions. No questions were asked to actively involve the child. Verbal support, provided by notes by the teacher, was provided. Responsiveness and Summarizing the needs of the child were rated as low. And the interview was judged to have a restricted scope (b). The child's mathematical development nevertheless increased from 183 (T1) to 229 (T2) for mathematical problem-solving ability and from 68 (T1) to 104 (T2) for arithmetic fluency. In addition, the child's math self-efficacy and self-concept increased while math anxiety decreased. No explicit math learning needs were identified during this dynamic math interview.

Figure 3 Some Examples of the Analyses of the Data Summarized in Table 1 Described in More Detail

Benefits of dynamic math interviews for identification of math learning needs

Identification of math learning needs was coded on the basis of explicit verbalisation by the child or verbalisation by the teacher with confirmation from the child (e.g., I need a ruler, I need to read the mathematical problem more thoroughly, check your answers). In 18 of the 19 analyzed dynamic math interviews (94.7%), math learning needs were explicitly identified; in one (*19*), they were not.

Table 2 Comparisons of Mathematical Development of Low Math Achievers in Control and Experimental Groups (between groups and within groups)

						Between
Children's mathematical development	Median Control Group (*n*=15) T1	Median Exp. Group (*n*=19) T1	Z	*p*	*g*	Median Control group (*n*=15) T2
1. Problem-solving	174	188	-1.321	.190	.651	203
2. Arithm. fluency	67	69	-. 608	.560	.035	83
2a. Addition	25	28	-1.026	319	.249	28
2b. Subtraction	18	17	- .052	.973	.014	18
2c. Multiplication	17	17	- .139	.891	.012	22
2d. Division	7	11	- .400	.706	.060	19
3. Self-efficacy	14	17	- .568	.580	.121	15
4. Self-concept	12	15.5	-1.854	.067	.596	15
5. Math anxiety	15	12.5	- .272	.789	.159	13

Note: * $p \leq .05$, ** $p \leq .01$; *** $p \leq .001$; T1 Start school year, T2 End school year. 1 = Mathematical problem-solving, 2 = Arithmetic fluency.

Dynamic math interviews and children's mathematical development

Addressing the second research question, to what extent the dynamic math interviews significantly promoted children's mathematical development, Table 2 shows the results of intervention-control comparisons. When we compared the means, standard deviations, and medians for the control versus experimental groups (for the medians, see Table 2), the results of a Wilcoxon signed-rank test *between groups* showed no significant effects on children's mathematical development. When combining a *p*-value of .065 with an effect size of .763 due to a small sample size, the results showed a trend towards an effect of the dynamic math interviews on mathematical problem-solving ability.

Significant *within groups* differences over time (T1-T2) were found for both the control and experimental groups on mathematical problem-solving and arithmetic fluency. The control group also increased significantly over time on addition and division skills; the experimental group on addition, subtraction, multiplication and division. No significant effects over time were found for math self-efficacy, math self-concept, or math anxiety.

groups				Within groups			
				Control Group T1 –T2		Exp. Group T1-T2	
Median Exp. Group (*n*=19) T2	Z	*p*	*g*	Z	*p*	Z	*p*
223	-1.859	.065	.763	-3.299	.001***	-3.765	.001***
95	- .746	.471	.126	-2.840	.005**	-2.878	.004**
29	- .766	.451	.243	-2.897	.004**	-2.988	.003**
22	- .869	.391	.221	-1.735	.083	-2.768	.006**
24	- .608	.560	.258	-1.052	.293	-2.092	.036*
17	- .869	.391	.287	-2.923	.003**	-2.251	.024*
17	-1.080	.286	.323	- .410	.682	-1.146	.252
17	- .835	.410	.286	-1.583	.113	- .192	.848
12	-1.183	.242	.425	- .199	.842	- .969	.333

Discussion

In this study, we investigated whether or not the use of dynamic math interviews could help teachers adequately identify the math learning needs of children and to what extent the use of dynamic math interviews could subsequently improve the mathematical development of children initially showing low mathematics achievement. Positive results were found for improved understanding of the math learning needs of children with initially low mathematics achievement. No significant differences were found between the experimental and control groups for mathematical development.

Eighteen of the nineteen interviews (94.7%) showed clear identification of specific math learning needs, such as the need for concrete visual-schematic representations, the need to read more carefully, the need to write down interim results, and the need to persevere and therefore not give up. It can be assumed that these needs and accompanying recommendations would not have been revealed using of standard testing. The conduct of a dynamic math interview allows the teacher to better appreciate the child's point of view, identify their specific math learning needs, and hence select suitable interventions (i.e., interventions which are within the child's zone of proximal development) (Lee & Johnston-Wilder, 2013).

The five teachers who demonstrated the highest levels of competence in the conduct of their dynamic math interviews also showed qualitatively good mathematics teaching behavior during the observed mathematics lessons. Nevertheless, there were teachers who showed high scores on teaching behavior but less than adequate dynamic math interviews and teachers who showed adequate dynamic math interviews but low teaching behavior scores. There may be, at most, an indication that math interviewing competence and mathematics teaching competence may somewhat be related, which corresponds to the findings of a previous study by Hoth et al. (2016).

The teacher professional development program used in combination with a practice period involving peer review and reflection on video-recorded dynamic math interview appear to have facilitated the teachers' ability to follow the dynamic math interview protocol, ask more open questions (among other things), and thereby better explore and understand child's mathematical knowledge, thinking, problem-solving procedures, experiences, emotions, and beliefs (Elliot et al., 2010; Empson & Jacobs, 2008; Ginsburg, 2009; Wright et al., 2006). The training of teachers to ask questions aimed at actively involving children in identification of their needs by asking solution-focused questions also enhanced the conduct of the dynamic math interviews (Bannink, 2010). At the start of the study, teachers were not familiar with such questions and their subsequent use appears to have contributed to the identification of a greater number of math learning needs (as seen in six dynamic math interviews).

It is striking that many of the teachers in our study spontaneously noticed children being able to solve a mathematical problem during the dynamic math interview which they previously could not solve. A calm but stimulating learning climate with a focus on the thorough reading of instructions and word mathematical problems, thinking out loud, and writing down interim steps in problem solution are examples of math learning needs determined during dynamic math interviews. These identified math learning needs supplement standardized test results.

Whether or not the dynamic math interviews had added value for the mathematical development of children with initially low mathematics achievement (our second research question) could not be answered positively: No significant differences between the experimental and control groups were found. As might be expected, the control and experimental groups both showed significant progress on mathematical problem-solving and arithmetic fluency. Regarding arithmetic fluency, the control group increased significantly over time on addition and division skills; the experimental group also on subtraction and multiplication. It is conceivable that a longer-term intervention is needed to show an impact on mathematical development. The second measure of mathematics achievement was administered shortly after the completion of the dynamic math interviews, so teacher might not have had sufficient time to master putting what they have learned into daily practice. A hint in this direction is the finding that mathematical problem-solving ability in the experimental group appeared to progress more than that in the control group. As a possible result of the dynamic math interview professional development program and practice, teachers may better recognize the complexity of mathematical problem-solving and what is required to effectively teach it.

The dynamic math interviews may have contributed to the ability of the teachers in our study to understand *why* some mathematics skills constitute a stumbling block for certain children and/or certain domains of mathematics. This information may have proved useful, in turn, for identifying just how they can better meet the needs of these children. In other words, the adequate conduct of dynamic math assessment in the form of a dynamic math interview appears to be particularly promising for identifying the specific math learning needs of individual children (also see Caffrey et al., 2008).

Study strengths, limitations, and directions for future research

A strength of the present research is the involvement of the same teachers in the control and experimental conditions (years 1 and 2). The involvement of the same teachers allowed us to control for variables which might otherwise influence the reliability of our results (e.g., possible cohort effects, teaching style). Another strength is the involvement of teachers and children coming from a variety of schools in the Netherlands, which suggests that our results are fairly representative. Another strength is that the video-recordings and observations were done in the real school setting and the dynamic math interviews conducted with children in their own school contexts.

We created what appears to be a useful teacher professional development program with the focus on dynamic math interviews. Furthermore, we developed a scripted tool for the conduct of dynamic math interviews that can presumably be used in all domains of mathematics and with all children. The tools proved reliable enough for more widespread use and examination on a larger scale. Furthermore, an analytic framework was clearly articulated and developed to facilitate the qualitative analyses of the dynamic math interviews conducted by the teachers. Further refinement of the framework is nevertheless needed. For example, adequacy of responding or, in other words, responding which is well-timed and allows the child to take advantage of the teacher's response was only scored as an overall impression within our analytic framework. More in-depth exploration and specification of teacher responding is thus needed (Empson & Jacobs, 2008).

Additional research is called for on the interrelations between math interviewing competence and mathematics teaching competence (and vice versa). We expect the proficient conduct of dynamic math interviews to help teachers identify the specific math learning needs of children and subsequently incorporate this information into their daily teaching practices to become better teachers. This will include, for example: more responsive listening and provision of suitable support, more attention to the mathematical problem-solving processes which children need to use and more involvement of children in determining and meeting their math learning needs (e.g., Deunk et al., 2018; Gersten et al., 2009).

A clear limitation on the present study is the relatively small sample size. This is nevertheless common in studies with detailed, qualitative coding of behavior and child-teacher interactions. But caution should be exercised when attempting to generalize the results to other settings, problems, and/or populations. Another possible limitation is that the last measurement was taken shortly after the conducted dynamic math interviews. An adjustment of the planning of the intervention over time is recommended.

The present study is a first attempt to analyze the adequacy and potential benefits of using dynamic math interviews with elementary school children (in this study: children known to have low mathematics achievement). Replication and expansion to include more teachers and a wider variety of children is therefore welcome.

Implications for practice

Dynamic math interviews proved useful for gaining insight into the mathematical thinking and problem-solving processes of children but also their math beliefs, emotions, fears, and the types of support needed in learning mathematics. With the competent use of a dynamic math interview, as found in the present study, teachers may be better able to attune the support which they provide to the individual child's zone of proximal development and thereby maximize the effectiveness of their efforts. It may nevertheless be the case that not only the introduction of a teacher professional development program and dynamic math interview practice are needed to foster a better recognition and understanding of the math learning needs of children today; it is possible that a more systemic implementation of dynamic assessment techniques is needed within the wider school context and learning community (Franke et al., 2001). In today's mathematics classrooms, children showing low mathematics achievement (or low achievement in general) require extra attention. The conduct of dynamic math interviews is a promising tool for providing the attention which is needed and thereby meeting the math learning needs of all children.

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Chapter 6

Summary and general discussion

Mathematics is a crucial but complex process with various factors influencing its learning and teaching. The research reported on here aimed to shed light on the roles of various child and teacher factors in the mathematical development of a large number of fourth grade children and to explore the effectiveness of using dynamic math interviews to identify and better meet their math learning needs. The two main research questions were thus as follows.

1) How can children's mathematical development, specifically their arithmetic fluency and mathematical problem-solving, be predicted from child and teacher factors?
2) To what extent does the use of dynamic math interviews facilitate the identification of the math learning needs of children, promote teachers' mathematics teaching and promote children's mathematics learning?

In this final chapter, the empirical results of the present research are summarized and discussed, the strengths and some possible limitations are pointed out, and some suggestions for future research are offered. In closing, the implications of the research outcomes for actual educational practice are discussed.

Child predictors of mathematical development

In Chapters 2 and 3, the outcomes are reported of the studies examining the predictive roles of cognitive factors, math-related beliefs, and math-related emotions on children's mathematics achievement by the end of grade 4 and their mathematical development during the course of grade 4.

With regard to the influences of specific *cognitive factors*, both the levels of arithmetic fluency and mathematical problem-solving achievement at the start of grade 4 contributed substantially to the children's mathematical development during fourth grade. This was the case for both of the studies presented in Chapters 2 and 3, respectively. And the finding is in line with what we expected on the basis of other research. Children's mathematical development is clearly facilitated when a sufficient foundation has been laid and the children thus have: an understanding of basic concepts, sufficient arithmetic fluency, a mastery of core calculation principles, an ability to identify and apply the operations necessary to solve mathematical problem (Andersson, 2008; Fuchs et al., 2016; Geary, 2004, 2011; Geary & Hoard, 2005). Findings in this research corroborate aspects of the hierarchical frameworks for mathematics in which is proposed that both domain-specific mathematical knowledge and more general cognitive processes (i.e., visuospatial and verbal updating, inhibition and shifting) underpin children's mathematical development (Cragg et al., 2017; Geary, 2004; Geary & Hoard, 2005). The relevance of prior mathematical knowledge and skills, in this research the entrance achievement level at the start of grade 4, has been confirmed.

With respect to the contribution of executive cognitive functioning and arithmetic fluency to their mathematical problem-solving *achievement*, the research in Chapter 3 showed arithmetic fluency, visuospatial and verbal updating to directly predict mathematical problem-solving at the end of fourth grade while inhibition and shifting did not. With regard to the *development* of mathematical problem-solving during the course of grade 4, inhibition and shifting indirectly contributed to this via arithmetic fluency while visuospatial and verbal updating did not, neither directly or indirectly. The level of arithmetic fluency at the start of grade 4 (i.e., achievement) plays a major role in both children's mathematical problem-solving at the end of grade 4 and its development during the course of grade 4.

With regard to *mathematical problem-solving achievement,* the visuospatial and verbal updating in the mathematical problem-solving of the children at the end of grade 4 was expected and found to be important (Cragg et al., 2017; Passolunghi & Pazzaglia, 2004; Zheng et al., 2011). A direct role of inhibition and shifting in the mathematical problem-solving of the children at the end of grade 4 was not found but has also not been frequently found in previous research (Jacob & Parkinson, 2015). The inclusion of visuospatial and verbal updating in the present and other research may account for this finding. When visuospatial and verbal updating are considered in addition to inhibition and shifting within the same study, visuospatial and verbal updating predominate in the prediction of mathematical problem-solving at the end of grade 4 — a finding in line with the results of a meta-analysis of previous studies conducted in this area (Friso-van den Bos et al., 2013). It is also possible, of course, that updating contributes to both inhibition and shifting and therefore precludes any direct influences for inhibition and shifting. This is in keeping with the outcomes of other research showing that the direct influences of inhibition and shifting on mathematics achievement can only be determined when measured independent of visuospatial and verbal updating (Bull & Lee, 2014).

With reference to *development in mathematical problem-solving* during the course of grade 4, the visuospatial and verbal updating did not contribute to mathematical problem-solving of the children. The influence of visuospatial and verbal updating declined during the course of grade 4 while the indirect influences of inhibition and shifting increased. Children must solve an increasingly wider variety of mathematical problems during fourth grade and thus increasingly more advanced, multi-step mathematical fact and word problems - both with and without pictures - calling for numerous and different calculations within the *same* problem. Better inhibition and shifting are thus required (Bull & Scerif, 2001; Cantin et al., 2016; Verschaffel et al., 2020). This is reflected in the findings of this research. Inhibition and shifting similarly contributed indirectly to the changes (i.e., development) in the children's mathematical problem-solving during the course of grade 4 via arithmetic fluency and after control for their mathematical problem-solving at the start of grade 4. This result presumably reflects the fact that more arithmetically fluent children have less of a need than less arithmetically fluent children to inhibit/suppress incorrect responses during their calculations. Arithmetically fluent children may also be better at switching from one calculation strategy to another and adapting existing strategies or known procedures as needed to solve a problem (Fuchs et al., 2006, 2016; Geary, 2011; Wiley & Jarosz, 2012).

The roles of children's *math-related beliefs and emotions* - math self-concept, math self-efficacy, and math anxiety - in their mathematical development are further described in Chapter 2.

Math self-concept predicted arithmetic fluency but not mathematical problem-solving. Math self-concept is presumably based on experiences with math in the past and thus more stable than math self-efficacy, which is - by definition - more future oriented and therefore malleable (Wolff et al., 2018). Furthermore, fourth grade children have accumulated greater experience with arithmetic fluency than advanced mathematical problem-solving and the strong association between math self-concept and arithmetic fluency thereby accounted for as well (Marsh et al., 2005; Weidinger et al., 2018). Math self-efficacy predicted neither arithmetic fluency nor mathematical problem-solving, which was unexpected. It is possible that math self-efficacy only becomes a significant predictor of math abilities later in development (i.e., after grade 4). Young children are less able than older children to judge their math performance and thus have less well-formed expectations for their performance and beliefs about whether they will succeed or not (Pajares & Miller, 1994). Also, an unexpected finding in the present research was that math anxiety did not predict any aspect of the children's fourth grade math performance. The explanation for this may lie in that children in Dutch elementary schools generally experience encouraging environments and therefore math anxiety did not play a predictive role in this research. Dutch elementary school children have generally been found to have sufficient self-confidence for the learning of mathematics (Hickendorff et al., 2017; *Inspectie van het Onderwijs*, 2021; Mullis et al., 2020).

To summarize, children's prior math knowledge and skill, defined as the level of arithmetic fluency and mathematical problem-solving ability at the start of grade 4, was found to be an important predictor of their mathematical abilities in the present research both during and at the end of grade 4. Both visuospatial and verbal updating were significant predictors for their mathematical problem-solving at the end of grade 4, while inhibition and shifting related indirectly to the development of their mathematical problem-solving ability during the course of grade 4 (via arithmetic fluency and after control for their mathematical problem-solving at the start of grade 4). With regard to the predictive roles of the children's math-related beliefs and emotions, only math self-concept played a predictive role but then for only the development of arithmetic fluency over the course of grade 4 and not for mathematical problem-solving at any point. Neither math self-efficacy nor math anxiety were found to be predictive.

Teacher predictors of mathematical development

In Chapter 2, the roles of various teacher factors in children's mathematical development were also considered: mathematics teaching behavior, mathematical knowledge for teaching knowledge, and mathematics teaching self-efficacy. In Chapter 4, the contributions of participation in a professional development intervention and practice period with the conduct of dynamic math interviews for the identification of children's math learning needs were examined.

Mathematics teaching behavior was found to be a negative predictor for the development of both children's arithmetic fluency and mathematical problem-solving ability (Chapter 2). This finding was quite unexpected and in contrast to the findings of studies showing positive effects of teaching behavior (e.g., classroom management, interactive mathematics lessons) on children's mathematics achievement (Muijs & Reynolds, 2002; Stronge et al., 2011). A possible explanation for the contradictory role found for mathematics teaching behavior in the present research may lie in the complexity of teaching mathematics (Ball et al., 2008). The teaching of mathematics requires a wide variety of skills: attunement of teaching behavior to math learning goals, adapting teaching behavior towards flexible use of textbook content, drawing of connections between underlying concepts and procedures, and selection of suitable representations for problems and domains (Ball et al., 2008; Hiebert & Grouws, 2007). On the basis of a meta-analysis conducted by Kyriakides et al. (2013), it has been recommended that choices that teachers make during their math teaching should always be well-considered and adopted from effective mathematics teaching approaches to obtain the best teaching results. Another possible explanation for the contradictory role observed for mathematics teaching behavior in the present research may be that the mathematical education standards in the Netherlands are quite high (*Inspectie van het Onderwijs*, 2021; Mullis, 2020).

In previous research, the specific aspects of mathematics teaching behavior responsible for the prediction of children's math success were not specified (Rockoff, 2004; Seidel & Schavelson, 2007). The results of the present research (Chapter 4) showed the professional development program and practice with dynamic math interviewing to improve the more advanced aspects of teaching behavior (i.e., use of activating learning, differentiation and adaptation of lessons, explicit teaching of learning strategies and math-specific strategies); only the less complex teaching behaviors did not improve (e.g., safe and stimulating learning climate, efficient classroom management). The teacher-child dialogue conducted as part of the dynamic math interviews and information obtained in these interviews presumably increased teachers' awareness of the individual child's math learning needs. In this connection, Stipek et al. (2001) found that teachers who focus largely on product (i.e., correct responding), achievement, and speed of problem-solving during mathematics lessons, teach in a largely prescriptive manner and follow textbooks quite strictly. In contrast, teachers who focus on the underlying understanding of children, their ability to make sense of mathematics, and their adoption of appropriate actions tend to carefully listen, observe, analyze errors, draw connections between ideas and concepts, and ask the right questions — the teachers show, in other words, more complex teaching behavior (Lester, 2013).

The *mathematical knowledge for teaching* of the teachers in the present research played a predictive role in the development of the children's mathematical problem-solving but not their arithmetic fluency during grade 4 (Chapter 2). In other research, Campbell et al. (2014) similarly found that teachers' perceived mathematical knowledge and their awareness of children's learning needs predicted the mathematics achievement of their students. For the teaching of mathematical problem-solving, Ball et al. (2008) have emphasized the importance of teachers' mathematical knowledge and knowing how to apply this effectively during daily teaching practice. For grade 4 mathematics instruction, Muijs and Reynolds (2002) indeed found teachers to perceive themselves as having more content knowledge and the necessary teaching skills for early mathematics education than for later instruction (e.g., fractions and proportions). The instrument used in the research reported on here (see Appendix B) asked teachers to rate their pedagogical content knowledge, subject matter knowledge, and specialized content knowledge for various domains of mathematics. Additional information provided by a more detailed (i.e., item-by-item) examination of the teachers' questionnaire responses showed more than 90% of the participating (Dutch) teachers to have high (i.e., "large to very large") beliefs about their own mathematical knowledge and the teaching of the various domains of mathematics — both before and after the intervention (i.e., participation in the dynamic math interview teacher professional development program and practice). However, when it comes to items such as offering concrete examples in the domain of ratios, fractions, percentages, and decimals or adopting different types of activities within the domain of geometry, the responses showed more than 20% of the teachers to have lower beliefs (i.e., "to some extent"). This finding suggests that teachers do not find the teaching of the more complex aspects of mathematics to be easy. The findings reported in Chapter 4 showed the teachers' self-perceptions of their mathematical knowledge for teaching to have increased, following participation in the professional development program. With the explanation and practice garnered with regard to the various aspects of dynamic math interviewing (e.g., asking questions to assess children's understanding and needs for clarification, appropriate interpretation of children's underlying thinking and reasoning) but also their interactions with other teachers during the professional development program, they may have strengthened their mathematical knowledge.

Teacher self-efficacy within the context of teaching mathematics showed no associations with children's arithmetic fluency and negatively correlated with children's mathematical problem-solving (Chapter 2). These differential findings were not completely in accordance with what was expected and clearly contradict prior research showing positive associations between teacher self-efficacy and children's mathematics achievement in general (Ashton & Web, 1986; Tella, 2008). In a review study, Klassen et al. (2011) further showed the associations between teacher self-efficacy and children's mathematics achievement to not be as strong as commonly assumed. Nevertheless, in a study by Bruce et al. (2010), increases in teacher math self-efficacy correlated positively with increases in student mathematics achievement with teacher self-efficacy improved by participation in a professional development program with a focus teaching through mathematical problem-solving. It is thus promising that the present intervention for the training of dynamic math interviewing promoted teachers' self-efficacy for the teaching of mathematics (Chapter 4). It could be the case that the intervention as described in Chapter 4 contributed to teachers' beliefs in their competences to the teaching of mathematics.

To summarize, for the predictive role of *teacher factors* in children's mathematical development, the results reported in Chapter 2 show mathematics teaching behavior to be an unexpectedly negative predictor for the development of both arithmetic fluency and mathematical problem-solving. The results reported in Chapter 4 show mathematics teaching behavior to improve following participation in the professional development program. Neither mathematical knowledge nor mathematics teaching self-efficacy predicted the development of the children's arithmetic fluency. For the development of the children's mathematical problem-solving over the course of grade 4, mathematical knowledge for teaching was found to be a positive predictor and mathematics teaching self-efficacy a negative predictor. The findings of negative associations for children's mathematical problem-solving with the mathematics teaching behavior and mathematics self-efficacy of teachers were unexpected and difficult but not impossible to explain. As reported in Chapter 4, all of the teacher factors increased following participation in the professional development program with a focus on dynamic math interviewing. Given that teachers who show high mathematics teaching self-efficacy and high mathematical knowledge for teaching are known to better prepare and adapt their mathematics instruction than teachers showing lower levels of self-efficacy and knowledge (Chang, 2015; Hill et al., 2008; Nurlu, 2015), these intervention results are valuable.

The facilitating role of dynamic math interviews

The second research question concerned the extent to which dynamic math interviews can be shown to facilitate the identification of math learning needs and promote teachers' mathematics teaching and thereby children's mathematics learning. In Chapter 4, the *quality of the dynamic math interviews* conducted by teachers before and after participation in a professional development program and practice with the conduct of dynamic math interview was examined in addition to aspects of their *mathematics teaching behavior, mathematical knowledge for teaching*, and *mathematics teaching self-efficacy*. The research addressing the question whether dynamic math interviews are an adequate way to identify children's math learning needs is described in Chapter 5.

The *teacher professional development program* in combination with an extended *practice* period was found to clearly foster better deployment of dynamic math interviews and more effective mathematics teaching practice on the part of the teachers participating in the present research (Chapter 4). This was apparent from improvement of quality of the dynamic math interviews and from positive effects on teacher factors.

The professional development program included the explanation of a scripted tool specifically developed for purposes of the present research and to help teachers with the conduct of interactive and process-oriented (i.e., dynamic) math interviews (Allsopp et al., 2008; Ginsburg, 2009; Lee & Johnston-Wilder, 2013). The active involvement of the child in their own mathematics learning stands central in these interviews. As part of the program, teaching involving dynamic math interviews was explained along with active, practice-based methods for encouraging children's mathematics learning. Articles concerned with the teaching of mathematics were read and discussed. Coherence was achieved by focusing on adapting math teaching on learning needs of children. The use of video was a core component of the professional development program. The use of examples of dynamic math interviews on video, individual feedback from the trainer on the videorecorded pretest and posttest math interview, and feedback from other teachers on videorecorded math interviews proved to be effective features of the program. This finding is in line with research showing the use of videorecorded own practices as part of teacher professional development programs to be highly effective (Borko et al., 2011; Heck et al., 2019; Tripp & Rich, 2012).

It was expected that the professional development program would be effective in light of the fact that it was designed with the characteristics of effective professional development clearly in mind (Desimone, 2009; Heck et al., 2019; Van Driel et al., 2012). Effectiveness was demonstrated by the effects of the program on not only the quality of the dynamic math interviews conducted by the teachers but also their positive evaluations of the training program (e.g., satisfaction scores, judgements of utility) (Chapter 4). It is sometimes suggested that professional development programs can be more effective when teachers from the same school participate in a given program (Porter et al., 2010). The evaluative remarks of the teachers who participated in the present research, however, suggest that, the participation of teachers from *different* schools was beneficial. The teachers did not know each other prior to the program, which made them curious about each other's experiences and differing school contexts. The collaboration with colleagues from the same grade also stimulated the teachers to exchange information on their practices, explain their instruction decisions, and share other ideas to develop a deeper understanding of children's mathematics learning and benefit their teaching as a result (Kazemi & Franke, 2004).

On the basis of the outcomes revealed by the application of an analytic framework specifically developed to evaluate elements judged to be critical for the identification of children's math learning needs, direct positive effects of the professional development program were found (Chapter 4). Compared to the pre-test math interviews, in post-test dynamic math interviews teachers asked significantly more questions about children's math experiences, beliefs, and emotions; they asked more questions about the children's reasoning and adopted problem-solving processes; they created a safer but also more stimulating classroom/learning climate; and they identified and stated the children's math learning needs more often. The teachers also focused their dynamic math interviews on more aspects of children's mathematical development (e.g., beliefs and emotions, solving processes) than prior to the start of the professional development program.

Despite these positive outcomes indicating the effectiveness of the professional development program, no effects were found for the following: questions aimed at active involvement of the child in identification of individual math learning needs; provision of support other than verbal; and posing of questions to determine the child's level of prior knowledge/understanding and thereby the adequacy of their knowledge and understanding. It is possible that the duration of the development program was too short to yield more widespread, positive effects (Garet et al., 2001). Illustrative in this light are the results reported in Chapter 5. When the quality of the dynamic math interviews conducted with children showing low mathematics achievement in particular (during the professional development practice period) was examined, the *majority* of the teachers asked more process- than product-oriented questions and they also actively involved the children in the identification of their individual math learning needs. Verbal support was still the most frequently utilized support. The research in Chapter 4 supports the notion that a professional development program based on the training characteristics known to be effective together with the offering of a scripted tool for the conduct of dynamic math interviews can clearly promote the quality of dynamic math interviews on several fronts. A more extended practice period may be called for as we all know that "practice makes perfect."

In 18 out of 19 dynamic math interviews was demonstrated that the math learning needs of children showing low mathematics achievement were identified (Chapter 5), dynamic math interviewing was shown to be effective. Such formative assessment clearly facilitated insight into the individual child's math learning needs and the adaptation of ongoing teaching and input to meet these needs. The use of dynamic math interviews has also been shown be a productive and welcome addition to standardized tests (Ginsburg, 2009; Veldhuis et al., 2013). For example, teachers obtained information about a child's zone of proximal development. The added value was also clearly the case in the present research (Chapters 4 and 5).

The second research question also concerned the extent to which dynamic math interviews promoted the identification of children's specific math learning needs and this information transferred to teachers' actual mathematics teaching and thereby enhanced their mathematical knowledge for teaching and mathematics teaching self-efficacy? The expectation was that the use of dynamic math interviews would allow the teacher to obtain an informed view of a child's understanding of mathematics and identify the child's math learning needs, on the one hand, and that this information would provide input for improved teaching, on the other hand (Allsopp et al., 2008; Carney et al., 2016). This was indeed found to be the case (Chapter 4).

The teachers in the present research clearly benefitted from the input provided by dynamic math interviews and put this information into daily teaching practice to provide more effective and adapted mathematics lessons. After training on the conduct of dynamic math interviews, activated learning was used more often; more differentiated and adapted lessons and general teaching strategies were employed; and more varied and adapted math-specific teaching strategies were used. No effects were found, however, for less advanced teaching behaviors that included the creation a safe but stimulating learning climate, efficient classroom management, and clarity of instruction. A dynamic classroom context that includes children with varying math learning needs nevertheless calls upon more advanced teacher competencies (Forgasz & Cheeseman, 2015; Porter et al., 2000). Teachers who are better able to assess children's mathematical knowledge and skills but also children's underlying thinking and planning can provide better adapted instruction and support (Hoth et al., 2016; Ketterlin-Geller & Yovanoff, 2009). And this was clearly found to be the case in the present research.

Teachers' perceptions of their mathematical knowledge for teaching improved with the professional development program on the use of dynamic math interviews and thereby their mathematics teaching self-efficacy as well. Beliefs and knowledge may interact to influence mathematics teaching behavior (Charalambous, 2015; Künsting et al., 2016; Wilkins, 2008). These findings are in keeping with other findings showing the mathematical knowledge for teaching and mathematics teaching self-efficacy to increase following participation in a professional development program focused on student thinking, problem-solving, and math-specific content knowledge (Carney et al., 2016).

Only limited mathematics learning effects were found for the influence of the conduct of dynamic math interviews with a sample of 19 children showing initially low mathematics achievement. Significant increases were found for the development of arithmetic fluency in the domains of subtraction and multiplication, but no effects for the development of the children's mathematical problem-solving, their math-related beliefs, or their math-related emotions. The learning of mathematics is obviously a long-term process requiring a solid foundation and extended practice (Ball et al., 2008; Lester, 2013). In the present research, the teachers conducted several interviews but only one interview per child. The influence of a teacher conducting a single dynamic math interview with a child already showing low mathematics achievement is thus limited (or nonexistent) but can be expected to increase (or at least occur) with repeated use.

To summarize, much of the specific knowledge and skills required for use of dynamic math interviews during mathematics teaching practice can be taught and enhanced via participation in a professional development program specifically designed for this purpose. Dynamic math interviews can be considerate an effective means for gaining insight into children's math learning needs and better understanding these needs. Moreover, dynamic math interviewing can improve teachers' mathematics teaching and thereby contribute to both their mathematical knowledge for teaching and mathematics teaching self-efficacy. The improvement of children's mathematical development with the use of dynamic math interviews has yet be demonstrated.

Strengths, limitations and directions for future research

The strength of this research lies in its longitudinal design. It covered two consecutive school years, involved the same teacher participants throughout the duration and reached across a variety of elementary schools spread across the Netherlands. A major strength is the large and representative sample size of children of grade 4. Furthermore, in this research several tools were created: a teacher professional development program focusing on dynamic math interviewing, a scripted tool to support the conduct of dynamic math interviewing in all domains of mathematics (Kaskens, 2016, 2018) and a framework to facilitate the qualitative analysis of the dynamic math interviews (Appendix C). Furthermore, a scale supplemented to the International Comparative Analysis of Learning and Teaching (Appendix A) and a measure for teachers' sense of their mathematical knowledge for teaching are developed the Teachers' Sense of Mathematical Knowledge for Teaching Questionnaire (Appendix B).

The quasi-experimental study design used in the present research had the advantage of involving the same teachers over time and therefore control for variables that might otherwise influence the reliability of the data (e.g., possible cohort effects and extended variability in teaching style could be ruled out). However, there was no control group in the present research. Replication with the inclusion of a control group and thus participation of a larger number of teachers is therefore recommended for the future.

Only quantitative measures were used to assess children's arithmetic fluency and mathematical problem-solving in the present research. Use of more process-oriented, qualitative measures (e.g., observation, analyzing of worked out strategies, think-alouds) might have provided greater insight into the approaches and strategies used by the children for a given task (Kotsopoulos & Lee, 2012; Ostad, 2000). Similarly for the assessment of teacher characteristics and competencies, the use of exclusively quantitative methods may not have captured all aspects or the richness of their mathematics teaching. Aspects of the interpersonal interaction between the teacher and child may have been missed (e.g., pay attention, appropriate responsiveness, type of feedback). The use of specific mathematics terminology by the teachers that is crucial to children's understanding and encourages children to correctly use the mathematical vocabulary, may have been missed. The manner in which the teacher responds when a child adopts an alternative approach to solve a given problem but also the teacher *actually* meeting (or not meeting) the math learning needs of the child may have been missed. In other words, the use of both quantitative and qualitative measurement instruments can be recommended for future research.

Finally, the last measurement of the teacher and child factors examined in the present research was shortly after the intervention practice period. For example, the timespan between the conduct of the interview and testing of the child, which varied between 2 to 6 weeks depending on the teachers' agenda was (too) short. The teaching of mathematics is known to be quite complicated and to require teachers to adapt their mathematics lessons to the — often quite divergent — needs of the children in their classrooms (Ball et al., 2008; Forgasz & Cheeseman, 2015).

Dynamic math interviews were shown to improve mathematics teaching behavior and anything else but not children's mathematics learning. Follow-up research is therefore recommended to examine the maintenance of the positive effects found for actual mathematics teaching behavior, mathematical knowledge for teaching and mathematics teaching self-efficacy but also the possibility of promoting children's mathematical development after all.

Conclusions and implications for actual practice

The research reported on here emphasizes the importance of establishing a solid mathematical foundation in the years leading up to elementary school grade 4 (see Byrnes & Wasik, 2009; Duncan et al., 2007; Hiebert & Grouws, 2007). Increased attention to the building of a solid mathematical foundation is therefore needed in actual teaching practice. This can be done by expanding, refining, and deepening children's conceptual understanding, factual knowledge and procedural skill. To do so, teachers could for example help children to understand which concepts are key, how to flexibly adapt previous experience to new transfer problems, offer various practice opportunities. Teachers can stimulate advanced mathematical problem-solving ability by helping children acquire the required skills, which may include: identification of relevant information and key words after the reading of a problem; devising a solution plan; consideration of alternative strategies; selection and application of most suitable strategies, operations, and algorithms; and learning to do all of this across a variety of contexts (Verschaffel et al., 2020). Achieving arithmetic fluency — for those who have not — and maintaining this requires teaching that is focused on not only drill-and-practice to improve speed and accuracy of basic arithmetic skills but also stimulation of children to identify underlying relationships, alternative strategies, and strategies in need of practice to attain and improve arithmetic fluency. A good balance between the acquisition of skills, rules, and procedures is essential for successful mathematics instruction.

To stimulate and improve children's mathematics learning and math-related thinking, it is crucial that teachers have a capacity to do the following, among other things: make carefully considered choices; adopt and apply elements from a variety of teaching approaches shown to be effective; and adapt their teaching to the identified learning needs of children (Kyriakides et al., 2013; Mitchell, 2015). They should also recognize that interventions aimed at improving executive functioning are best conducted in relation to domain-specific goals (Jacob & Parkinson, 2015).

Given that a clear association was found in the present research between children's math self-concept and arithmetic fluency, we can conclude that it is important for children to be given plenty of opportunities *early in their development* for the learning of mathematics. Only then can the elementary school child feel sufficiently confident and thus comfortable to tackle the challenge of mathematical problem-solving. Math self-concept is more past-oriented than — for instance — math self-efficacy (Wolff et al., 2018) and should therefore be recognized as a critical factor in children's mathematics education.

A point related to the above is that fourth grade children have greater experience with arithmetic than with mathematical problem-solving. It is thus important that performing domain-specific interventions to promote successful mathematical problem-solving and enhance (or maintain) math self-concept be part of the mathematics curriculum and teaching. Praise and immediate, targeted, concrete, and otherwise confidence-building feedback are crucial for this (O'Mara-Eves et al., 2006).

The recognition and understanding of children's varying math learning needs is a prerequisite for adapted and differentiated mathematics instructions and thus part and parcel of successful mathematics education (Charalambous, 2015; *Inspectie van het Onderwijs*, 2019). Formative assessment is therefore called for to gain insight into children's mathematical thinking, understanding, development, and needs (Ginsburg, 2009; Veldhuis et al., 2013). Dynamic math interviewing is a promising assessment tool for teachers and other professionals to thus use to assess children's prior knowledge and skills, thinking and problem-solving processes, and their math-related beliefs, emotions, experiences and needs. With the information gained by the teacher in such a dialogue with the child, teachers can attune their interventions to within the child's so-called zone of proximal development and thus maximize the effectiveness of the support they provide. Dynamic math interviewing thus supplements standardized testing. It has to be noted that a dynamic math interview can be applied in an interactive dialogue between a teacher and one child, but also a small group of children.

The outcomes of the present research also showed that teachers clearly benefit from a professional development program specifically designed to promote the implementation and use of dynamic math interviews. Such training on a more widespread and possibly standard basis for teachers should therefore be considered in the future.

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Samenvatting
(Summary in Dutch)

Inleiding

Leren rekenen is belangrijk voor het goed kunnen functioneren in de maatschappij. In de eerste jaren op de basisschool wordt een belangrijke basis gelegd: kinderen ontwikkelen getalbegrip, leren belangrijke rekenbegrippen en bouwen stapsgewijs rekenwiskundige concepten en procedures op. Geleidelijk groeit de beheersing van basisvaardigheden: optellen, aftrekken, vermenigvuldigen en delen. Vanaf ongeveer groep 6 neemt de complexiteit van het rekenen[1] op school toe. Leerlingen krijgen dan complexere tekstuele contextopgaven voorgelegd, waarin diverse berekeningen moeten worden toegepast. Daarnaast krijgen ze nieuwe rekenonderdelen, zoals breuken en procenten, aangeboden.

De verschillen in de rekenwiskundige ontwikkeling tussen leerlingen zijn groot en kunnen worden voorspeld vanuit cognitieve factoren (bijvoorbeeld redeneren, geheugen), domeinspecifieke inzichten (zoals begrijpen wat een breuk is), en kennis en vaardigheden (zoals vlot kunnen optellen en aftrekken tot 20). Ook competentiebeleving en emoties van leerlingen ten aanzien van rekenen kunnen van invloed zijn op hun rekenwiskundige ontwikkeling. Elke schooldag weer staan leraren voor de uitdaging om het rekenwiskunde-onderwijs af te stemmen op de verschillen tussen leerlingen. Om dat te realiseren is het nodig dat leraren inzicht hebben in de onderwijsbehoeften van leerlingen bij het leren rekenen.

Het voeren van rekengesprekken (‘*dynamic math interviews*’) is een aanpak waarbij leraren en andere onderwijsprofessionals (zoals intern begeleiders, remedial teachers, orthopedagogen) op interactieve, procesgerichte wijze met de leerling in gesprek gaan met als doel de onderwijsbehoeften te achterhalen met actieve betrokkenheid van de leerling. Via gerichte wederkerige vragen probeert de leraar te ontrafelen wat een leerling nodig heeft om zich verder te kunnen ontwikkelen op rekengebied. Tijdens een rekengesprek krijgt de leraar zicht op het niveau van beheersing van diverse rekenonderdelen, op de mate waarin de leerling beschikt over onderliggende/voorwaardelijke kennis en vaardigheden, en op denk- en oplossingsprocessen van de leerling. Bovendien komt de leraar idealiter meer te weten over aan rekenen gerelateerde ervaringen, overtuigingen en emoties. De leerling heeft een actieve rol tijdens het rekengesprek doordat de leraar de leerling uitnodigt mee te denken over doelen, mogelijke oplossingen en/of werkzame aanpakken. Het voeren van rekengesprekken vereist specifieke kennis en vaardigheden van de leraar, waarvan verwacht wordt dat deze kunnen worden ontwikkeld door professionalisering gericht op rekengespreksvoering.

Veel onderzoek is al gedaan naar de invloed van leerling- en leraarfactoren op de rekenwiskundige ontwikkeling, maar relatief weinig onderzoek combineert de verschillende factoren in een en hetzelfde design. Bovendien wordt zelden onderscheid gemaakt tussen geautomatiseerde kennis van de basisvaardigheden (dat wil zeggen, vlot en accuraat kunnen optellen, aftrekken, vermenigvuldigen en delen) en het kunnen oplossen van rekenwiskundige problemen. Met rekenwiskundige problemen wordt in deze dissertatie bedoeld, rekenopgaven waarin rekenwiskundige notaties en tekst en/of illustraties worden gebruikt, opgaven zoals deze in de reguliere Nederlandse rekenwiskundemethoden veel voorkomen[2]. Het oplossen van rekenwiskundige problemen vereist conceptueel begrip, vlot berekeningen kunnen uitvoeren, en specifieke kennis en vaardigheden. Denk bijvoorbeeld aan het snel kunnen oproepen van opgeslagen kennis uit het geheugen, de juiste informatie uit een opgave kunnen halen die nodig is om tot een oplossing te komen, en het flexibel kunnen toepassen van diverse strategieën.

De eerste twee studies van deze dissertatie (hoofdstukken 2 en 3) zijn gericht op het ontrafelen welke specifieke rol bepaalde leerling- en leraarfactoren spelen in de rekenwiskundige ontwikkeling van leerlingen in groep 6. In groep 6 wordt de transitie gemaakt naar meer complexere rekenwiskundige leerdoelen. Gezien het belang van leren rekenen is het zinvol om meer inzicht te krijgen in de rol van leerling- en leraarfactoren. De laatste twee studies (hoofstukken 4 en 5) zijn gericht op rekengespreksvoering. Voor zover ons bekend is nog niet empirisch onderzocht of rekengesprekken voeren een effectieve aanpak is om onderwijsbehoeften bij rekenen te achterhalen en of de aanpak bijdraagt aan verbetering van rekenwiskunde-onderwijs dat beter is afgestemd op leerlingen.

Noot: [1]) Overal waar rekenen wordt gebruikt wordt rekenen-wiskunde bedoeld.

[2]) Rekenwiskundige probleemoplossingsvaardigheden worden gedefinieerd als het oplossen van niet-routinematige rekenwiskundige problemen, waarbij kinderen worden uitgedaagd om eigen oplossingswijzen te bedenken en toe te passen (Polya, 1957; Doorman et al., 2007). In deze dissertatie wordt uitgegaan van rekenopgaven waarin rekenwiskundige notaties en tekst en/of illustraties worden gebruikt.

Doorman, M., Drijvers, P., Dekker, T., Van den Heuvel-Panhuizen, M., De Lange, J., & Wijers, M. (2007). Problem solving as a challenge for mathematics education in the Netherlands. ZDM, 39, 405-418. https://doi.org/10.1007/s11858-007-0043-2. Polya, G. (1957). How to solve it: A new aspect of mathematical method. 2nd ed. Princeton University Press.

Leerlingfactoren

Op de rekenontwikkeling zijn zowel *cognitieve factoren* (zoals geheugen en domeinspecifieke kennis) als *overtuigingen* en *emoties* van de leerling ten aanzien van rekenen van invloed. In deze dissertatie is onderzocht wat de specifieke rol is van het niveau van geautomatiseerde basiskennis en rekenwiskundige probleemoplossingsvaardigheden waarover leerlingen begin groep 6 beschikken. Bovendien is in een studie (hoofdstuk 3) onderzocht welke rol *executieve functies* spelen, de reguleringsfuncties die denkprocessen in het brein aansturen. Daarbij zijn *visuospatieel* en *verbaal updaten*, *inhibitie,* en *shifting* onderscheiden. Bij updating gaat het om het opslaan, bewerken en verwerken van opgeslagen informatie in het werkgeheugen als er nieuwe informatie binnenkomt. Bij visuospatiële updating gaat het om visueel-ruimtelijk aangeboden informatie, ofwel het zien en verwerken van waarnemingen in de ruimte, bijvoorbeeld het onthouden van de locatie van een blokje. Bij verbale updating gaat om opslaan, bewerken en verwerken van gesproken of geschreven talig aangeboden informatie, zoals het onthouden van tussenantwoorden tijdens het oplossen van een rekenopgave. Bij inhibitie gaat het om het kunnen onderdrukken van niet adequate respons, bijvoorbeeld bij een opgave als 4 + 5 niet doortellen (de automatische respons om de telrij op te zeggen onderdrukken). Bij shifting gaat het om het vermogen om flexibel te kunnen wisselen tussen bewerkingen en/of strategieën als dit bij rekentaken nodig is. Dit is bijvoorbeeld van belang bij rekenopgaven waarin diverse bewerkingen moeten worden uitgevoerd (denk aan een opgave als 102-98 en dan niet een uitgebreide aftrekhandeling uitvoeren, maar bedenken dat bij 98 vier opgeteld kan worden om 102 te krijgen, of een breukopgave waarbij tafelkennis moet worden toegepast en vervolgens nog moet worden opgeteld).

Als het gaat om overtuigingen en emoties, zijn in dit onderzoek drie aspecten betrokken die een rol lijken te spelen in de rekenwiskundige ontwikkeling van leerlingen, *math self-concept*, *math self-efficacy* en *math anxiety*. Math self-concept betreft het *zelfbeeld* van leerlingen ten aanzien van rekenen. Het gaat om het eigen oordeel van de leerling over de mate waarin hij/zij goed denkt te zijn in rekenen volgens de eigen standaarden, het zelfbeeld met betrekking tot rekenen. Math self-efficacy kan worden omschreven als *competentiebeleving*, de perceptie van de eigen competentie om rekentaken met succes te kunnen voltooien. Leerlingen met hoge competentiegevoelens zijn meer dan anderen geneigd om moeilijke taken als een uitdaging te zien, hebben een sterk commitment met gestelde leerdoelen en hebben een grote bereidheid om nieuwe strategieën uit te proberen. Math anxiety (*rekenangst*) is een emotie, een negatieve reactie op rekenen. Zo zijn er leerlingen die blokkeren op het moment dat ze een vel met sommen voor zich zien of die stressgevoelens ervaren op het moment dat de rekenles begint. Dit kan er bijvoorbeeld toe leiden dat leerlingen rekentaken gaan vermijden.

Leraarfactoren

Leren rekenen en dus ook het rekenwiskunde-onderwijs heeft betrekking op lange termijn leerprocessen. Leraren kunnen bijdragen aan de rekenwiskundige ontwikkeling van leerlingen. Dit kan bijvoorbeeld door interactief en activerend lesgeven, het gebruik van diverse materialen en representaties, door relaties te leggen tussen verschillende onderdelen van het rekenen en door bewuste keuzes te maken op welke manier het onderwijs kan worden afgestemd op leerlingen. Het afstemmen van het rekenwiskunde-onderwijs op de verschillen tussen leerlingen vraagt de nodige kennis en kunde van leraren. Zo moeten ze in staat zijn om vooruitgang te monitoren, inzicht hebben in wat leerlingen nodig hebben voor de verdere ontwikkeling en beschikken over voldoende vakspecifieke kennis en vaardigheden.

In deze dissertatie zijn drie factoren onderscheiden die op basis van eerder onderzoek als belangrijke aspecten worden gezien in relatie tot het geven van rekenwiskunde-onderwijs. Dat zijn *leraarhandelen* tijdens de rekenles, waarbij rekenlessen zijn geobserveerd aan de hand van een observatie-instrument. Verder is de inschatting van leraren als het gaat om hun *vakspecifieke kennis* van het (onderwijzen van) rekenen-wiskunde betrokken. Een derde aspect dat is betrokken is *competentiebeleving*, de mate waarin leraren zichzelf competent vinden ten aanzien van het onderwijzen van het vak rekenen-wiskunde.

De rol van rekengesprekken

Leraren hebben te maken met leerlingen die onderling verschillen in hun rekenwiskundige ontwikkeling. Om goed afgestemd rekenwiskunde-onderwijs te kunnen bieden is het nodig om te weten wat leerlingen nodig hebben om zich goed te kunnen ontwikkelen op rekengebied. Doorgaans wordt hiertoe vooral gebruik gemaakt van gestandaardiseerde, genormeerde, productgerichte toetsen. Steeds meer onderwijsprofessionals zijn ervan overtuigd dat voor het in kaart brengen van de onderwijsbehoeften van leerlingen formatieve beoordelingsvormen nodig zijn die procesgericht zijn en informatie kunnen bieden over *hoe* de leerlingen verder in hun ontwikkeling kunnen worden gestimuleerd. Rekengesprekken zouden daarin kunnen voorzien, zodat leraren die informatie kunnen benutten in het dagelijks handelen, bijvoorbeeld welke voorwaardelijke kennis nog aandacht behoeft, wat voor soort instructie en werkvormen bevorderend zijn, welke uitdagende taken kunnen worden aangeboden. Aangezien er nog geen uitgewerkt hulpmiddel voorhanden was voor het voeren van rekengesprekken, is dit hulpmiddel ten behoeve van dit onderzoek ontwikkeld.

Het voeren van rekengesprekken vereist specifieke kennis en vaardigheden, zoals het kunnen creëren van een veilig en prettig klimaat, goede vragen stellen en responsief reageren, en het kunnen inzetten van de nodige vakspecifieke kennis om goed te kunnen volgen hoe leerlingen denken en redeneren om daarop vervolgens goed aan te kunnen sluiten.

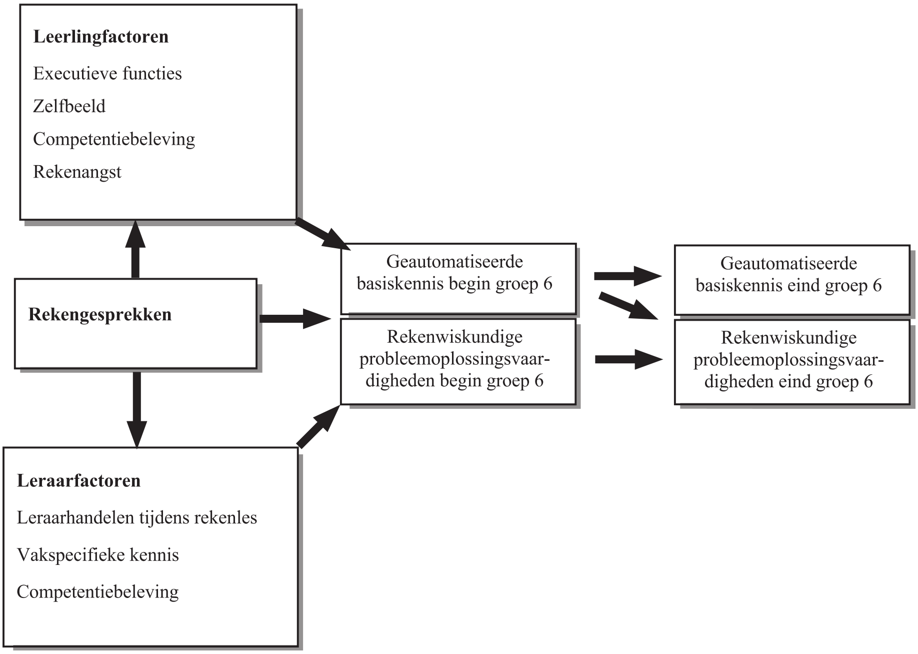
In de laatste twee studies van deze dissertatie (hoofstukken 4 en 5) staat rekengespreksvoering centraal.

De vragen die in dit onderzoek worden beantwoord zijn de volgende:

1. Hoe kan de rekenwiskundige ontwikkeling van leerlingen, in het bijzonder ten aanzien van geautomatiseerde basiskennis en rekenwiskundige probleemoplossingsvaardigheden, worden voorspeld vanuit leerling- en leraarfactoren?
2. In hoeverre helpt het voeren van rekengesprekken a) het achterhalen van de onderwijsbehoeften van leerlingen bij rekenen, b) het rekenwiskunde-onderwijs door leraren, en c) de rekenwiskundige ontwikkeling van leerlingen?

In de eerste twee studies (hoofdstukken 2 en 3) staat de eerste onderzoeksvraag centraal. In een longitudinaal onderzoeksdesign is nagegaan in hoeverre de rekenontwikkeling van leerlingen in groep 6, voor zowel geautomatiseerde basiskennis alsmede rekenwiskundige probleemoplossingsvaardigheden, te voorspellen is vanuit de volgende leerlingfactoren: het rekenniveau aan het begin van groep 6, zelfbeeld en competentiebeleving ten aanzien van rekenen, mate van rekenangst en executieve vaardigheden. Ook is onderzocht in hoeverre de volgende leraarfactoren voorspellend zijn voor de rekenontwikkeling van leerlingen: leraarhandelen tijdens de rekenles, kennis van het (onderwijzen van) rekenen-wiskunde en de competentiebeleving ten aanzien van het onderwijzen van rekenen-wiskunde.

In de laatste twee quasi-experimentele studies (hoofstukken 4 en 5) staat de tweede onderzoeksvraag centraal. De interventie in dit onderzoek bestaat uit een professionaliseringsprogramma en een daaropvolgende oefenperiode in rekengespreksvoering. Onderzocht is wat het effect is van de interventie op de kwaliteit van de gevoerde rekengesprekken en of er sprake is van effecten op de leraarfactoren en op de rekenontwikkeling van leerlingen. Voor een overzicht van alle componenten in relatie tot rekenen, die in deze dissertatie zijn onderzocht, zie Figuur 1.



Figuur 1 Een overzicht van de in dit onderzoek betrokken componenten, in relatie tot rekenen

Het onderzoek was longitudinaal en vond plaats in twee achtereenvolgende schooljaren. Dezelfde leraren van groep 6 deden twee jaar lang mee. Daarbij moet worden opgemerkt dat in het eerste jaar 31 leraren zijn gestart. Als gevolg van onder andere ziekte, zwangerschap en wisseling van baan vielen acht leraren uit en hebben uiteindelijk 23 leraren twee jaar lang geparticipeerd. De groep leerlingen in jaar 1 is een andere groep leerlingen dan in jaar 2, de leraar kreeg immers in het tweede jaar een nieuwe groep leerlingen. In het eerste jaar zijn metingen uitgevoerd, maar vond geen interventie plaats. Dat jaar is de controle-conditie. Het tweede jaar waarin de interventie is uitgevoerd, betreft de experimentele conditie. In Figuur 2 staat het onderzoeksdesign.

Ter ondersteuning van het ontwikkelen en uitbreiden van de nodige kennis en vaardigheden is een professionaliseringprogramma ontworpen, gebaseerd op wat uit de literatuur bekend is over effectieve kenmerken. De kennis en vaardigheden die nodig zijn voor adequate rekengespreksvoering, vormden de kern van het programma. Nadat de leraren de bijeenkomsten hebben bijgewoond en in diezelfde periode ook hebben geoefend met het voeren van rekengesprekken, hebben ze in de maanden daarna nog meer geoefend met het voeren van rekengesprekken. De leraren hebben in deze tijdsspanne met zes leerlingen van verschillende rekenniveaus uit hun eigen klas een rekengesprek gevoerd.

<table>
<tr><th colspan="8">Schooljaar 1: controle groep</th></tr>
<tr><td>aug-sep</td><td>okt</td><td colspan="3">nov-mid feb</td><td>feb</td><td>maart-half juni</td><td>juni</td></tr>
<tr><td>Meting 1, jaar 1</td><td colspan="6">Regulier rekenwiskunde-onderwijs</td><td>Meting 2, jaar 1</td></tr>
<tr><th colspan="8">Schooljaar 2: experimentele groep</th></tr>
<tr><td>Meting 1, jaar 2</td><td>Individuele feedback op een reken-gesprek</td><td>Pre test</td><td>Professio-naliserings-program-ma</td><td>Post test</td><td>Individuele feedback op een reken-gesprek</td><td>Oefen-periode</td><td>Meting 2, jaar 2</td></tr>
</table>

Figuur 2 Onderzoeksdesign

Samenvatting van de resultaten

Allereerst worden de resultaten beschreven behorend bij de onderzoeksvraag: Hoe kan de rekenwiskundige ontwikkeling van leerlingen, in het bijzonder ten aanzien van geautomatiseerde basiskennis en rekenwiskundige probleemoplossingsvaardigheden, worden voorspeld vanuit leerling- en leraarfactoren? (hoofdstukken 2 en 3).

In hoofdstuk 2 zijn de resultaten beschreven van een longitudinale studie naar de specifieke voorspellende rollen van zowel leerling- als leraarfactoren voor de rekenontwikkeling van leerlingen van groep 6. Daarbij zijn geautomatiseerde basiskennis en rekenwiskundige probleemoplossingsvaardigheden onderscheiden. De leerlingfactoren die in deze studie zijn gemeten zijn rekenprestaties, zelfbeeld en competentiebeleving ten aanzien van rekenen, en de mate van rekenangst. De leraarfactoren zijn (geobserveerd) leraarhandelen tijdens de rekenles, de eigen inschatting van vakspecifieke kennis en de competentiebeleving ten aanzien van het (onderwijzen van) rekenen-wiskunde. Data van 610 leerlingen en 31 leraren van groep

6 zijn verzameld en via multilevel-analyses geanalyseerd. Daarbij heeft controle voor non-verbaal redeneervermogen plaatsgevonden, omdat dit een factor kan zijn die een onderliggende rol speelt bij probleemoplossingsvaardigheden.

De resultaten laten zien dat de ontwikkeling van geautomatiseerde basiskennis bij leerlingen van groep 6 mede wordt voorspeld vanuit het niveau van begin groep 6 en vanuit het zelfbeeld van leerlingen ten aanzien van rekenen. Echter, het leraarhandelen tijdens de rekenwiskundeles is negatief gerelateerd aan de ontwikkeling van geautomatiseerde basiskennis. Dus leraren met een hogere score op het handelen tijdens de rekenles hebben in hun klas leerlingen met een minder goede ontwikkeling van geautomatiseerde basisvaardigheden.

De ontwikkeling van rekenwiskundige probleemoplossingsvaardigheden wordt mede voorspeld vanuit het niveau van leerlingen aan het begin groep 6. Ook vakspecifieke kennis is mede voorspellend voor de ontwikkeling van rekenwiskundige probleemoplossingsvaardigheden van leerlingen. Het leraarhandelen tijdens de rekenwiskundeles en de competentiebeleving zijn daarentegen negatief gerelateerd. Dat betekent dat leraren met een hoge score op het leraarhandelen en hun eigen competentie ten aanzien van het lesgeven hoog beoordelen, leerlingen in hun klas hebben met een minder goede ontwikkeling van rekenwiskundige probleemoplossingsvaardigheden.

Zowel ten aanzien van de ontwikkeling van geautomatiseerde basisvaardigheden alsmede de ontwikkeling van rekenwiskundige probleemoplossingsvaardigheden van leerlingen, was de verwachting dat het *leraarhandelen* tijdens de rekenles een belangrijke rol zou spelen in de rekenontwikkeling van leerlingen. Deze verwachting was gebaseerd op resultaten van eerdere studies. Een mogelijke verklaring voor de onverwachte resultaten in deze studie zou kunnen zijn dat het onderwijzen van rekenen-wiskunde complex is en veel kennis en vaardigheden van leraren vereist, zoals kennis over doelen en didactiek, het goed kunnen uitleggen van bepaalde oplossingswijzen, het aanbieden van oplossingsstrategieën op verschillende abstractieniveaus, het bevorderen van zelfvertrouwen, het benutten van betekenisvolle situaties, en materialen en representaties passend kunnen inzetten.

De door leraren zelf ingeschatte *vakspecifieke kennis* blijkt in deze studie mede een voorspeller te zijn voor de ontwikkeling van de probleemoplossingsvaardigheden van leerlingen. Mogelijk zijn leraren er zich van bewust dat er meer specifieke kennis van de leraar vereist is als het gaat om de wat complexere aspecten van het rekenwiskunde-onderwijs, zoals het ondersteunen van het rekenleerproces van kinderen bij het oplossen van rekenwiskunde-opgaven met een hoger abstractieniveau.

Voor wat betreft de *competentiebeleving ten aanzien van het onderwijzen van rekenen* zijn de resultaten niet overeenkomstig de verwachtingen: geen significante relatie met de ontwikkeling van geautomatiseerde basisvaardigheden en een negatieve relatie met de ontwikkeling van rekenwiskundige probleemoplossingsvaardigheden. Diverse studies laten wel positieve relaties zien tussen competentiebeleving van leraren en de rekenprestaties van leerlingen. Wat mogelijk een rol heeft gespeeld is dat minder competente leraren niet goed in staat zijn om hun incompetentie te herkennen als het gaat om het onderwijzen van rekenen-wiskunde, wat kan leiden tot overschatting.

Ten aanzien van de *cognitieve leerlingfactoren* blijkt een stevige basis van rekenwiskundige kennis en vaardigheden van leerlingen, die wordt opgebouwd in de jaren voorafgaand aan groep 6, van groot belang te zijn voor de rekenontwikkeling in groep 6. Dit geldt zowel voor de geautomatiseerde basiskennis als voor het oplossen van rekenwiskundige problemen. Deze bevinding is overeenkomstig de verwachtingen gebaseerd op eerdere studies, maar het belang ervan voor de ontwikkeling van leerlingen in groep 6 is in dit onderzoek bekrachtigd.

Voor wat betreft de *overtuigingen en emoties* van leerlingen ten aanzien van rekenen, is het zelfbeeld ten aanzien van rekenen voorspellend voor de ontwikkeling van geautomatiseerde basiskennis. Dit is niet aangetoond voor de competentiebeleving. Dit heeft mogelijk te maken met het feit dat het zelfbeeld gebaseerd wordt op ervaringen met rekenen in het verleden (dat wil zeggen, de jaren voorafgaand aan groep 6) en dat competentiebeleving een grotere rol gaat spelen in latere leerjaren. Uit eerdere studies is gebleken dat jongere kinderen minder goed kunnen inschatten hoe competent ze zijn op rekengebied. Op basis van de resultaten van deze studie lijkt het zo te zijn dat het kunnen inschatten van de competentiebeleving door leerlingen pas op latere leeftijd verwacht kan worden, dus mogelijk pas een voorspellende waarde heeft als ze ouder zijn dan leerlingen van groep 6. Rekenangst komt evenmin als voorspeller naar voren. Mogelijk is dit te verklaren uit het gegeven dat de rekenleeromgeving op de participerende scholen over het algemeen bemoedigend en ondersteunend is, waardoor deze negatieve emotie in deze studie niet als factor van betekenis naar voren komt.

In hoofdstuk 3 zijn de resultaten gerapporteerd van de tweede longitudinale studie waarin de rol van executieve vaardigheden op de rekenontwikkeling van leerlingen is onderzocht. Ook hierbij is gecontroleerd voor non-verbaal redeneervermogen. In deze studie zijn data van 458 leerlingen uit de totale leerlingpopulatie van 1062 leerlingen over de twee schooljaren heen verzameld. Binnen deze groep is sprake van een evenwichtige verdeling van laagpresterende, gemiddeld presterende en hoog presterende leerlingen (gebaseerd op het rekenniveau).

Allereerst is met een regressie-analyse de rol van geautomatiseerde basiskennis begin groep 6 en executieve vaardigheden op het *prestatieniveau* van rekenwiskundige probleeem-oplossingsvaardigheden eind groep 6 onderzocht. Daaruit blijkt dat geautomatiseerde basiskennis aan het begin van groep 6 een belangrijke voorspeller is. Van de vier executieve vaardigheden zijn visuospatieel en verbaal updaten ook voorspellers voor het prestatie*niveau* van probleeemoplossingsvaardigheden eind groep 6, inhibitie en shifting evenwel niet. Dit kan mogelijk te maken hebben met het gegeven dat updating ook als variabele in de studie betrokken is. In eerdere studies waarin naast updating ook andere executieve vaardigheden zijn betrokken, blijkt updating de sterkste voorspellende waarde te hebben.

Vervolgens is een mediatie-analyse uitgevoerd om te onderzoeken welke factoren van directe en welke van indirecte invloed zijn op de *ontwikkeling* van probleeemoplossingsvaardigheden in groep 6. Daarbij zijn de executieve vaardigheden als onafhankelijke variabelen, het niveau van geautomatiseerde basiskennis begin groep 6 als mediërende variabele, en probleemoplossingsvaardigheden begin groep 6 als covariaat betrokken. Uit de resultaten blijkt dat de directe invloed van de geautomatiseerde basiskennis en probleemoplossingsvaardigheden aan het begin van groep 6 van invloed is op de ontwikkeling van deze kennis en vaardigheden gedurende groep 6. Inhibitie en shifting blijken gedurende groep 6 van toenemende invloed te zijn op de ontwikkeling van probleemoplossingsvaardigheden (via geautomatiseerde basiskennis aan het begin van groep 6), terwijl de rol van visuospatieel en verbaal updaten afneemt. Kennelijk doet de toenemende complexiteit van rekenwiskundige problemen in groep 6 een groter beroep op inhibitie en shifting. Denk bijvoorbeeld aan het grotere beroep dat bij complexe contextopgaven wordt gedaan om aandacht voor irrelevante informatie te kunnen onderdrukken en flexibel te kunnen schakelen van de ene bewerking naar de andere binnen eenzelfde opgave. Bij het oplossen van rekenwiskundige probleemoplossingsvaardigheden blijken groep 6 leerlingen bovendien profijt te hebben van een voldoende geautomatiseerde basiskennis. In deze studie is bevestigd - overeenkomstig de verwachtingen gebaseerd op eerdere studies en de bevindingen van onze eerste studie - hoe belangrijk het voor leerlingen is, dat ze in de jaren voorafgaand aan groep 6 een stevige basis van rekenwiskundige kennis en vaardigheden opbouwen.

De resultaten van de tweede onderzoeksvraag 'In hoeverre helpt het voeren van rekengesprekken a) het achterhalen van de onderwijsbehoeften van leerlingen bij rekenen, b) het rekenwiskunde-onderwijs door leraren, en c) de rekenwiskundige ontwikkeling van leerlingen?'

In hoofdstuk 4 zijn de resultaten beschreven van de quasi-experimentele studie naar effecten van rekengespreksvoering op leraarfactoren. In totaal zijn 23 leraren betrokken in deze studie, allen hebben ze beide jaren aan het onderzoek deelgenomen. De interventie die in deze studie centraal staat is een professionaliseringsprogramma, gevolgd door een periode van oefening in het voeren van rekengesprekken met leerlingen. Door middel van een voor- en nameting is nagegaan wat het effect is van het programma op de kwaliteit van de rekengesprekken, die voorafgaand aan de professionalisering en aan het eind van de professionalisering op video zijn opgenomen. Uit de resultaten komt naar voren dat het *professionaliseringsprogramma* effect heeft op bepaalde aspecten die bijdragen aan de kwaliteit van de rekengesprekken. De leraren stellen meer vragen gericht op ervaringen en beleving ten aanzien van (leren) rekenen, op het redeneer- en oplossingsproces van een leerling, ze creëren een veiliger en stimulerender klimaat tijdens het rekengesprek, en ze vatten vaker -in samenspraak met de leerling- de onderwijsbehoeften van de leerling op rekengebied samen. In vergelijking met de voormeting waren de rekengesprekken tijdens de nameting op meer verschillende aspecten gericht die een rol spelen bij de rekenontwikkeling (bijvoorbeeld niet alleen op de rekenkennis, maar ook op ervaringen met rekenen). Op de volgende aspecten is geen aanzienlijk verschil tussen voor- en nameting gevonden: het actief betrekken van de leerling bij het nadenken over de onderwijsbehoeften en het nagaan van de voorwaardelijke kennis en vaardigheden van de leerling. Verder bieden de leraren meer verschillende vormen van hulp aan bij de nameting (bijvoorbeeld structuur bieden, complexiteit verminderen, modellen en schema's gebruiken), maar in vergelijking met de voormeting is dit verschil niet aanzienlijk.

Om de effecten van de *interventie* (deelname aan het professionaliseringsprogramma gevolgd door de oefenperiode) op leraren te onderzoeken zijn de leraarfactoren gemeten op vier meetmomenten: drie voorafgaand aan de interventie en één na de interventie aan het eind van het schooljaar. Vervolgens zijn door middel van herhaalde variantie-analyses (ANOVA) de verschillen tussen deze vier meetmomenten nagegaan, gevolgd door post hoc tests om te bepalen waar de verschillen tussen het derde en vierde meetmoment zich voordeden. Daarna is het verschil bepaald tussen de baseline (eerste drie meetmomenten) en tussen het derde en vierde meetmoment. Hierbij zijn de effecten op het leraarhandelen tijdens de rekenles, de ingeschatte vakspecifieke kennis en de competentiebeleving ten aanzien van het (onderwijzen van) rekenen nagegaan. De effecten op deze drie leraarfactoren waren significant.

Opvallend is dat er grote effecten van de interventie zijn op de meer complexere aspecten van het leraarhandelen tijdens de rekenles:

activerend leren, differentiëren en afstemmen, en het onderwijzen van leerstrategieën en rekenspecifieke onderwijsstrategieën. Op de minder complexe aspecten was sprake van geringe effecten: veilig en stimulerend leerklimaat, klassenmanagement en het geven van een duidelijke instructie. De interventie heeft bovendien effect op de door de leraren zelf ingeschatte vakspecifieke kennis en competentiebeleving.

De rekengesprekken die zowel tijdens het professionaliseringsprogramma als in de daaropvolgende oefenperiode zijn gevoerd, leveren informatie op over de rekenontwikkeling van de leerlingen. De verkregen input tijdens de rekengesprekken blijkt van invloed te zijn geweest op het leraarhandelen tijdens de rekenles, en heeft bovendien positief bijgedragen aan de door de leraren zelf ingeschatte vakspecifieke kennis en competentiebeleving. Er lijkt een wisselwerking te zijn tussen het ontwikkelen van kennis en vaardigheden van leraren door middel van de interventie en hun percepties.

In hoofdstuk 5 zijn de resultaten gerapporteerd van een studie naar de effectiviteit van rekengesprekken als aanpak voor het achterhalen van onderwijsbehoeften van laagpresterende leerlingen en in hoeverre er effecten waren op leerlingfactoren. Met ‘laagpresterend’ gaat het om leerlingen die beneden het 20ste percentiel scoorden op de Cito rekentoets. Er zijn 19 rekengesprekken kwalitatief geanalyseerd. Om te onderzoeken of het rekengesprek invloed heeft op de leerlingfactoren (rekenontwikkeling, zelfbeeld, competentiebeleving en mate van rekenangst), is vanwege de kleine onderzoeksgroep een non-parametrische meting (Wilcoxon signed-rank test) toegepast, waarbij de verschillen tussen de experimentele groep en de controlegroep zijn vergeleken.

Uit de resultaten blijkt dat in 18 van de 19 rekengesprekken de onderwijsbehoeften van de leerlingen ten aanzien van rekenen-wiskunde worden achterhaald, de hoofddoelstelling van rekengespreksvoering. Voorbeelden van onderwijsbehoeften zijn de behoefte aan extra instructie, zorgvuldiger lezen, gebruik van materialen, het noteren van tussenstappen of tussenuitkomsten, samenwerken met bepaalde leerlingen, oefenen van bepaalde vaardigheden, vragen durven stellen tijdens de rekenles, niet opgeven. De analyse ten aanzien van kwaliteitsaspecten van rekengespreksvoering laat zien dat de meerderheid van de leraren tijdens de oefenperiode aanzienlijke vooruitgang boekt. Daar heeft de professionalisering en oefening mogelijk aan bijgedragen. Voor zes leraren kan worden geconcludeerd dat op alle aspecten sprake is van een rekengesprek van goede kwaliteit. Daarbij is het van belang om op te merken dat kwaliteitsaspecten in samenhang tot elkaar moeten worden bekeken. Verder is geanalyseerd of er sprake was van een samenhang tussen de kwaliteit van rekengespreksvoering en het geobserveerde leraarhandelen tijdens de rekenles, maar die samenhang is niet eenduidig: er waren leraren die hoog scoorden op leraarhandelen, maar minder adequate rekengesprekken voerden en vice versa.

Een beduidend verschil tussen de gemeten leerlingfactoren van de experimentele en de controlegroep is er alleen ten aanzien van de basisvaardigheden aftrekken en vermenigvuldigen als aspecten van geautomatiseerde basiskennis. Verder zijn er geen aanzienlijke verschillen ten aanzien van leerlingfactoren naar voren gekomen. Mogelijk hangt dit samen met het gegeven dat de rekengesprekken kort voor de eindmetingen zijn gevoerd en dat er met de betreffende leerling slechts één rekengesprek is gevoerd.

Conclusie en aanbevelingen voor de praktijk

De bevindingen van het onderzoek benadrukken het belang van een solide rekenwiskundige basis in de jaren voorafgaand aan groep 6. Veel aandacht besteden aan het uitbreiden, verfijnen en verdiepen van het conceptueel begrip, feitelijke kennis en procedurele vaardigheden is dus van belang. Executieve functies komen in deze dissertatie ook naar voren als een factor van betekenis in de rekenontwikkeling van leerlingen. Als het gaat om het bevorderen van executieve functies blijkt uit eerdere studies, dat interventies met name effectief zijn als deze direct in relatie staan tot rekenspecifieke doelen tijdens de rekenles. Denk bijvoorbeeld aan het demonstreren dat de leerling de opgave eerst zorgvuldig moet lezen, relevante informatie uit een opgave moet halen en moet nadenken over de aanpak alvorens de berekening uit te voeren. Of leerlingen ondersteunen door een complex probleem in hanteerbare delen op te splitsen.

Verder is een duidelijke relatie aangetoond tussen het zelfbeeld van leerlingen ten aanzien van rekenen en geautomatiseerde basiskennis. Het bekrachtigt het belang om juist in de leerjaren voorafgaand aan groep 6 optimale kansen te bieden om te leren rekenen. Daarmee kan worden bijdragen aan voldoende zelfvertrouwen ten aanzien van rekenen en aan meer vertrouwen bij het oplossen van rekenwiskundige problemen, ook als deze wat complexer worden. Aangezien leerlingen in groep 6 namelijk meer ervaring hebben opgedaan met het verwerven van basisvaardigheden in vergelijking met het oplossen van complexe rekenwiskunde-opgaven, kan het zinvol zijn om specifieke interventies uit te voeren die gericht zijn op het ontwikkelen van probleemoplossingsvaardigheden.

In deze dissertatie zijn ook onverwachte resultaten gevonden. De door leraren zelf ingeschatte vakspecifieke kennis is een voorspeller gebleken voor de ontwikkeling van de probleemoplossingsvaardigheden, maar het leraarhandelen tijdens de rekenles en de competentiebeleving van leraren blijken geen voorspellers voor de rekenontwikkeling van leerlingen. Het onderwijzen van rekenen-wiskunde in groep 6 lijkt complex te zijn en vraagt veel kennis en vaardigheden van leraren. Om de rekenwiskundige ontwikkeling van leerlingen te bevorderen en af te stemmen op de onderwijsbehoeften van leerlingen, is het noodzakelijk dat leraren weloverwogen keuzes maken tijdens voorbereiding en uitvoering van de rekenles.

Om de verschillende onderwijsbehoeften van kinderen te achterhalen is in deze dissertatie gebruikgemaakt van rekengesprekken. Rekengesprekken zijn een vorm van formatief assessment, waarmee inzicht kan worden gekregen in rekenniveau, begrip en inzicht, voorwaardelijke kennis en vaardigheden, rekenontwikkeling, redeneer- en oplossingsprocessen, beleving, emoties en overtuigingen, en behoeften van leerlingen met betrekking tot rekenen. Op basis van deze dissertatie is de conclusie dat rekengespreksvoering een veelbelovende aanpak lijkt te zijn die door leraren en andere onderwijsprofessionals kan worden ingezet om de onderwijsbehoeften te achterhalen, aanvullend op standaard toetsen. Effecten op leerlingresultaten zijn niet aangetoond. Leren rekenen is een lange termijn proces en effecten op de betreffende leerling zijn misschien pas te verwachten na een langere tijd en/ of na meerdere rekengesprekken. Leraren profiteren van een professionaliseringsprogramma en oefening, gericht op het adequaat leren voeren van rekengesprekken. Effecten zijn gedemonstreerd op leraarhandelen tijdens de rekenles, inschatting van de vakspecifieke kennis en competentiebeleving. De informatie die leraren verkrijgen tijdens rekengesprekken met verschillende leerlingen, kan bevorderend zijn voor de rekenwiskundeles en kan leraren ondersteunen in het toepassen van interventies binnen de zone van naaste ontwikkeling van leerlingen. De opbrengsten van dit onderzoek kunnen worden benut in opleidingen en in de onderwijspraktijk en als zodanig bijdragen aan gefundeerd rekenwiskunde-onderwijs.

Dankwoord (Acknowledgements)

In september 2014 liep ik de marathon van Berlijn. Dat betekende gedisciplineerd trainen, focus houden en voldoende rust nemen om uiteindelijk zonder blessures te finishen. Toen de dag daar was, nam ik me voor om er vooral van proberen te genieten, temeer omdat ik wist dat ik maar een keer in mijn leven een marathon zou mogen lopen. Het was een dag om nooit te vergeten. In 2016 kreeg ik de kans om een promotieonderzoek te starten. Terugblikkend betekende dat het begin van een training voor een mentaal-cognitieve marathon. Ook deze training vereiste gedisciplineerd werken, focus houden en voldoende ontspannen en fit blijven om de eindstreep te halen. Ik verwachtte dat ik er veel van zou leren en mijn streven was dat ik met dit promotieonderzoek een bijdrage zou kunnen leveren aan het onderwijs. Een traject om nooit te vergeten. Het is volbracht en daar hebben velen direct of indirect aan bijgedragen.

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Als het even zou kunnen, wil ik een staande ovatie voor de leraren en alle groep 6 leerlingen! Zonder jullie enorme inzet was het nooit gelukt om alle data te verzamelen. Ik realiseer me dat deelname aan dit onderzoek veel van jullie heeft gevraagd. Wat heb ik genoten van het bijwonen van de rekenlessen, van de individuele testmomenten met de kinderen en de bijeenkomsten met de leraren. Tijdens het transcriberen van de rekengesprekken heb ik vaak beseft hoe belangrijk het is dat we meer met leerlingen praten in plaats van over. Jullie hebben laten zien hoe je dat kunt realiseren en dat is top.

Alle personen die een rol hebben gespeeld als het gaat om rekengesprekken: Jos Kienhuis, Dianne Roerdink, Sui Lin Goei en Henk Logtenberg die de eerste zaadjes hebben geplant. Alle studenten van de Pabo, Master Educational Needs en leraren van diverse scholen (in het bijzonder Erik Kuiper en Lisanne Oosterveen-Visse). Door jullie was ik steeds weer geïnspireerd en gemotiveerd om een aanpak te ontwikkelen en te verfijnen die uiteindelijk tot een praktisch raamwerk heeft geleid. Inmiddels uitgebracht als boek dat voor het voeren van rekengesprekken in alle onderwijssoorten kan worden gebruikt.

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About the author

Jarise Kaskens was born on June 21st, 1965, in Veghel (the Netherlands). She obtained her secondary school degree (VWO) from the Mgr. Zwijsen College in Veghel. In 1983, she enrolled at Utrecht University to study Orthopedagogiek (Clinical Child, Family and Education Studies). She specialized in educational psychology and her thesis focused on mathematics. After obtaining a master's degree with honors in 1988, she began work as an assistant researcher at Utrecht University, combined with a part-time position as a school psychologist and teacher trainer. From 1992 to 2008 she worked as an educational consultant at the School Advice Center (SAC)/Eduniek in Utrecht. During that time, Jarise worked in a variety of roles, such as educational advisor, school psychologist, teacher trainer, project leader, video coach, educational developer and policy advisor. The subjects she focused on were mathematics education and early childhood education. Between 2008 and 2011 she worked as a managing consultant at the Pedagogical Study and Educational Advice Center (CPS), and focused on mathematics education, early childhood education, and reading education. She participated in several research and development projects. Since 2011 she has been a lecturer (hogeschoolhoofddocent) in the department of Movement and Education at Windesheim University of Applied Sciences, where she has worked in a variety of roles. Currently, she also has several different responsibilities. She works as a lecturer and student coach for the master's degree programs in Educational Needs and Learning and Innovation. She is project manager of the 'Teacher roles, didactics and pedagogical relationship' Learning Lab and is involved in the 'Meaningful and Inclusive Learning Environments' research group (lectoraat) led by dr. Sui Lin Goei. She participates in research projects involving mathematics, lesson study, and inclusive education.

Jarise received a doctoral grant from the Dutch Organization for Scientific Research NWO for her PhD proposal in 2016, which was awarded by prof. dr. Jet Bussemaker, former minister of Education, Culture, and Science. In the same year, she started her PhD project at the Behavioural Science Institute at Radboud University under the supervision of prof. dr. Ludo Verhoeven, prof. dr. Eliane Segers, prof. dr. Hans van Luit (Utrecht University) and dr. Sui Lin Goei (Windesheim). During this project, she developed a teacher professional development program and some new instruments. Jarise has written many articles for professional teaching journals, several books and has published several scientific journal articles. During her career, she has given presentations and workshops at various conferences (conferences aimed at primary, secondary and vocational education as well as research conferences). Her aim is to make connections between science, practice and policy.

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Appendices

Appendix A contains the Scale for Mathematics Teaching Strategies supplemented to The International Comparative Analysis of Learning and Teaching. Appendix B includes the Teachers' Sense of Mathematical Knowledge for Teaching Questionnaire (TSMKTQ). Appendix C contains the Analytical Framework to facilitate the qualitative analysis of the dynamic math interviews and in Appendix D examples of parts of the dynamic math interviews are presented.

Appendix A: Scale for Mathematics Teaching Strategies Supplemented to The International Comparative Analysis of Learning and Teaching (Kaskens, Segers, Goei, Van Luit, & Verhoeven, 2018).

Given that the ICALT is not math-specific, a scale of eight items was created (S). This validated scale for mathematics teaching strategies is supplemented to The International Comparative Analysis of Learning and Teaching (ICALT; Van de Grift 2007; Van der Lans et al., 2018).

ICALT Uitbreiding Rekenspecifieke schaal

Schoolnaam: **Naam leerkracht:**	**Datum observatie (dd-mm-jjjj):** **Naam observator:**
	Groep: **Aantal leerlingen:**

Rekenspecifieke uitbreiding op de ICALT – Lesobervatieformulier: evalueren van pedagogisch-didactisch handelen van leraren

Niveau Omcirkel het gewenste oordeel:
1 = overwegend zwak 2 = meer zwak dan sterk
3 = meer sterk dan zwak 4 = overwegend sterk

Gezien
Omcirkel het gewenste antwoord:
0 = nee, dat heb ik niet waargenomen
1 = ja, dat heb ik waargenomen

Indicator:	De leraar...	Niveau	Voorbeelden van goede praktijk. De leraar	Gezien
Hoort bij indicator: **Afstemmen op verschillen**	**36...maakt gebruik van informeel handelen met concreet materiaal (doen, ervaren, zien gebeuren)**	**1 2 3 4**	*...laat leerlingen handelen met concreet materiaal (bijv. appel verdelen in vier stukken; uitzoeken hoeveel minipakjes vruchtensap in 1 liter maatbeker passen)*	*0 1*
	37...maakt gebruik van concrete representaties (afbeeldingen van echte objecten en situaties)	**1 2 3 4**	*...zet foto's, illustraties in die de realistische situatie representeren (bijv. foto van een in acht stukken gesneden pizza; illustratie van een dashboard met benzinemeter met pijl op ¾ vol)*	*0 1*
	38...zet abstracte en schematische representaties in (modellen en diagrammen)	**1 2 3 4**	*...zet rekenwiskundige denkmodellen in, zoals lege getallenlijn, verhoudingstabel (bijv. cirkeldiagram inzetten om verdeling van percentages aan te geven; getallenlijn van 0 tot 1 gebruiken bij plaatsen van 0,8)*	*0 1*
	39...zet het formele niveau in (symbolisch niveau, mentale operaties, kale opgaven en talige rekenopgaven)	**1 2 3 4**	*...laat formele berekeningen uitvoeren (bijv. oplossen van kale opgaven of talige contextopgaven)* *...laat leerlingen rekenen met symbolen (bijv. ½, +)*	*0 1* *0 1*

	40…legt verbinding tussen handelings niveaus	**1 2 3 4**	*…schakelt tussen de handelingsniveaus (bijv. van afbeelding naar model en weer van model naar afbeelding)* *…maakt expliciet wat de relatie is tussen het ene en het andere handelingsniveau (bijv. met imitatiegeld bedragen samenstellen en vervolgens bedragen samenstellen waarbij de bedragen in een positieschema worden genoteerd)*	*0 1* *0 1*
Hoort bij indicator: **Leerstrate-gieen aanleren**	**41…besteedt aandacht aan de fase Plannen van het drieslagmodel**	**1 2 3 4**	*…laat leerlingen bij een context een som/bewerking bedenken* *…laat leerlingen bij een kale som een verhaal of tekening bedenken* *…stimuleert leerlingen betekenis te verlenen aan de getallen in relatie tot de context* *…zet leerlingen aan om relevante informatie uit de opgave te halen* *…zet leerlingen aan de informatie te ordenen (overlap met 27)* *…stimuleert leerlingen om een plan van aanpak te bedenken alvorens te gaan rekenen*	*0 1* *0 1* *0 1* *0 1* *0 1* *0 1*
	42…besteedt aandacht aan de fase Uitvoeren van het drieslagmodel **N.B. Deze overlapt met item 32**	**1 2 3 4**	*…heeft aandacht voor het oplossingsproces door vragen te stellen naar de wijze waarop leerlingen een opgave hebben opgelost (zie item 32)*	*0 1*
	43…besteedt aandacht aan de fase Reflecteren van het drieslagmodel **N.B. Overlap met item 29 (en enigszins met 31 en 32)**	**1 2 3 4**	*… stimuleert de leerlingen na te gaan of het antwoord kan kloppen (overlap met item 31)* *…besteedt aandacht aan wat het antwoord (het getal) betekent* *…vraagt leerlingen na te denken over de gebruikte oplossingsstrategie (bijv. handig of niet, kan het korter?) (overlap met item 32)*	*0 1* *0 1* *0 1*

Appendix B: Teachers' Sense of Mathematical Knowledge for Teaching Questionnaire (TSMKTQ) - Dutch version (Kaskens, Segers, Goei, Verhoeven, & Van Luit, 2018)

The Teachers' Sense of Mathematical Knowledge for Teaching Questionnaire is an online questionnaire constructed and validated for the present dissertation and focuses on teachers' pedagogical content knowledge, subject matter knowledge or specialized content knowledge. This self-assessment is related to the so-called 'Mathematics knowledge base for primary preservice education' (Van Zanten et al., 2009). This knowledge base prescribes the mathematical knowledge preservice teachers should master before graduation and is assessed by a nationwide test for preservice teachers. The instrument could be useful to encourage preservice ànd inservice teachers' reflection on this aspect of mathematics teaching and within professional development programs and teacher education.

Wat fijn dat u deze lijst voor ons wilt invullen!

Met deze lijst willen wij in beeld krijgen hoe leerkrachten zichzelf inschatten ten aanzien van domeinspecifieke vakkennis.
Het invullen van de lijst kost ongeveer 15 minuten. U kunt tussentijds stoppen en op een ander moment doorgaan.
De lijst is als volgt opgebouwd:
Deel 1 bestaat uit 5 items en gaat over het eigen vaardigheidsniveau;
Deel 2 bestaat uit 10 items en betreft het handelen van leerkrachten bij rekenen;
Deel 3 bestaat uit 23 items en heeft betrekking op kennis voor het onderwijzen van rekenen.

De gegevens worden anoniem verwerkt.

	In zeer geringe mate	In enige mate	In ruime mate	In zeer sterke mate
Deel 1: onderstaande vijf items gaan over uw eigen vaardigheidsniveau. 1 staat voor de inschatting dat u in zeer geringe mate beschikt over ...; 4 staat voor de inschatting dat u in zeer sterke mate beschikt over .... Schat in: de mate waarin u beschikt over rekenvaardigheid en gecijferdheid				
1. ..... op minimaal **3F** niveau. Toelichting: Indien u meer wilt weten over referentieniveau 3F, zie http://www.taalenrekenen.nl/ref_niveaus_rekenen/uitwerkingen/uitgelegd/				
2. ..... op het domein **Hele getallen**.				
3. ..... op het domein **Verhoudingen, procenten, breuken en kommagetallen**.				
4. ..... rekenvaardigheid en gecijferdheid op het domein **Meten**.				
5. .....rekenvaardigheid en gecijferdheid op het domein **Meetkunde**.				

	In zeer geringe mate	In enige mate	In ruime mate	In zeer sterke mate
Deel 2: de volgende tien items hebben betrekking op het handelen van leerkrachten ten aanzien van oplossingsprocessen en niveauverhoging. Schat in: de mate waarin u....				
6. ..... aan rekenen-wiskunde betekenis kunt geven voor leerlingen. Toelichting: U maakt gebruik van de realiteit en de actualiteit om rekenen betekenisvol te maken. U probeert bijvoorbeeld samen met de leerlingen fouten in een grafiek in de media te ontdekken.				
7. ...... oplossingsprocessen en niveauverhoging bij leerlingen kunt realiseren.				
8. ..... rekenfouten kunt begrijpen en een foutenanalyse kunt uitvoeren.				
9. ..... foutief of (nog) niet formeel gebruik van rekentaal opmerkt en kunt corrigeren.				
10. ..... redeneringen van leerlingen bij het oplossen van rekenopgaven kunt volgen en doorgronden				
11. ..... bij rekenopgaven meerdere alternatieve oplossingsmanieren kunt gebruiken.				
12. ..... bij veel voorkomende oplossingsstrategieën zowel denkstappen kunt toevoegen als verkortingen kunt aangeven.				
13. ..... van oplossingsmanieren kunt beoordelen in hoeverre deze perspectief bieden in het licht van langlopende rekenleerprocessen.				
14. ..... oplossingsmanieren op verschillende abstractieniveaus kunt aanbieden, afgestemd op leerlingen, en daarbij streeft naar een hoger abstractieniveau. Toelichting: Concreet handelen met materiaal is van een lager abstractieniveau dan een denkmodel gebruiken. Bijvoorbeeld: een verhoudingstabel gaan gebruiken als leerlingen het inzicht hebben dat een verhouding een vergelijking aangeeft van aantallen die naar voren komen in een bepaalde situatie, zoals afstand en tijd.				
15. .....bij leerlingen een positieve attitude en zelfvertrouwen ten aanzien van rekenen kunt bevorderen.				

	In zeer geringe mate	In enige mate	In ruime mate	In zeer sterke mate
Deel 3: alle items van dit gedeelte gaan over de mate waarin u over kennis beschikt voor het onderwijzen van rekenen-wiskunde. Schat in: de mate waarin u over kennis beschikt voor het onderwijzen van..... Toelichting: U zet uw kennis van rekenen in bij het onderwijzen en ondersteunen van het leren rekenen. U beheerst de opbouw van de leerlijnen en tussendoelen. Verder beheerst u didactische kennis die het leren op de basisschool op gang brengt, ondersteunt en stimuleert. Denk hierbij aan kennis over betekenisvolle contexten, modellen en schema's. Deze kennis past u toe om adaptief en diagnosticerend rekenwiskunde-onderwijs te realiseren. Bijvoorbeeld: weten dat de strategie 'rijgen' ondersteund wordt met de kralenstang en getallenlijn.				
16. ..... rekenen-wiskunde op het domein **Hele getallen**.				
16a. ..... de ontwikkeling van tellen en getalbegrip.				
16b. ..... het automatiseren van het optellen en aftrekken tot 10 en tot 20.				
16c. ..... het leren optellen en aftrekken tot 100 en verder.				
16d. ..... het leren vermenigvuldigen en delen.				
16e. ..... het leren schattend rekenen en hoofdrekenen.				
16f. ..... verschillende manieren waarop leerlingen de standaardprocedures en cijferalgoritmes kunnen leren (bijvoorbeeld kolomsgewijs rekenen en cijferen).				
16g. ..... het leren hanteren van de rekenmachine en het gebruik van de rekenmachine als onderzoeksmiddel en als rekenhulpmiddel				
17. ..... rekenen-wiskunde op het domein **Verhoudingen, procenten, breuken en kommagetallen.**				
17a. ..... specifieke verschijningsvormen van verhoudingen, procenten, breuken en kommagetallen en hoe deze kunnen worden ingezet ten behoeve van begripsvorming. Bijvoorbeeld korting als verschijningsvorm van procenten.				

17b. ..... contextsituaties die leerlingen uitlokken noties te ontwikkelen over de specifieke rekenwiskundige aard van verhoudingen, procenten, breuken en kommagetallen. Bijvoorbeeld recept voor 2 personen vertalen naar recept voor 8 personen.				
17c. ..... specifieke modellen voor en representaties van verhoudingen, procenten, breuken en kommagetallen en deze in de lespraktijk kunnen inzetten. Bijvoorbeeld de kans op zon is 30%; de verhoudingstabel inzetten voor het rekenen en redeneren met verhoudingen.				
17d. ..... verschillende concretiseringen en oplossingswijzen om leerlingen te helpen bij moeilijkheden die zich kunnen voordoen bij het leren van verhoudingen, procenten, breuken en kommagetallen. Bijvoorbeeld: helpen door flexibel te wisselen tussen verschillende concretiseringen en oplossingswijzen. Zoals het berekenen van 20% korting door het bepalen van het vijfde deel, de 1% regel toe te passen, te rekenen via de 10%, te rekenen met een kommagetal (0,20 x ..) en dit te visualiseren met een strook, of de stappen doorlopen aan de hand van een verhoudingstabel.				

	In zeer geringe mate	In enige mate	In ruime mate	In zeer sterke mate
Vervolg Deel 3: alle items van dit gedeelte gaan over de mate waarin u over kennis beschikt voor het onderwijzen van rekenen-wiskunde. **Schat in: de mate waarin u over kennis beschikt voor het onderwijzen van.....**				
18. ..... rekenen-wiskunde op het domein **Meten.**				
18a. ..... de opbouw van de leerlijn meten, waaronder het leren van het metriek stelsel door leerlingen.				
18b. ..... situaties waarin voor leerlingen herkenbare meetgetallen naar voren komen. Bijvoorbeeld de eigen groei van de leerling in centimeters.				
18c. ..... referentiematen bij standaardmaten. Bijvoorbeeld een pak suiker weegt een kilo.				
19. ..... rekenen-wiskunde op het domein **Meetkunde.**				

19a. ….. de verschillende soorten meetkundige activiteiten: Oriëntatie in de ruimte; Viseren en projecteren; Transformeren; Construeren; Visualiseren en representeren. Bijvoorbeeld verschuiven, draaien en spiegelen van figuren hoort bij transformeren. Het beredeneren welke informatie nodig is om een bouwsel goed te kunnen bouwen hoort bij construeren.				
19b. ….. activiteiten en situaties die meetkundige activiteiten uitlokken. En weten hoe u leerlingen kunt aanzetten tot meetkundige redeneringen op een voor de leerling passend niveau. Bijvoorbeeld het lokaliseren van de school op een plattegrond en de route van huis naar school beschrijven.				

Hartelijk dank voor de medewerking!

Appendix C: Analytical Framework

To explore the adequacy of a dynamic math interview, the following aspects of the dynamic math interviews can be analyzed.

1. *Ratio open to closed questions posed by teacher.* Open questions are assumed to elicit greater information and therefore preferred over closed questions. At the start of the dynamic math interview, closed questions may nevertheless be more suitable for the purpose to establish trust or to check the teacher has understood the child correctly. By asking in-depth questions (e.g., What did you mean by that?), the teacher can gain more information or clarity (Delfos, 2001; Ginsburg, 1997). The proportion open questions should be higher than the proportion closed questions.
2. *Questions focused on child's math experiences, beliefs, and emotions.* With the intention of a wider scope of a dynamic math interview, the teacher can also ask questions focused on child's math experiences, beliefs, and emotions. What kinds of mathematical problems do you find easy/hard? What in the mathematics lesson should change/not change? How does it feel when you cannot solve a problem? (Allsopp et al., 2008; Bannink, 2010; Ginsburg, 1997). The proportion of total number of questions with focus on child's math experiences, beliefs, and emotions is counted.
3. *Questions focused on child's thinking and problem-solving processes*. These questions help gain insight into what the child understands and they do not understand. How did you solve this problem? Tell me. The teacher can obtain an explanation for why the child does not understand things or cannot complete the problem correctly (e.g., Allsopp et al., 2008; Ginsburg, 1997, 2009). The proportion of total number of questions with focus on child's thinking and problem-solving processes is counted.
4. *Questions to check the child knows the right answer.* With these questions the teacher can gain information about mathematics achievement levels and mastery of skills. The attainment of process information as opposed to product (i.e., outcome) information should nevertheless prevail for the dynamic math interview to have added value near standardized tests (Franke et al., 2001; Van Luit, 2019). The proportion of total number of questions with focus on checking the child knows correct answer is counted.
5. *Questions to identify math learning needs by actively eliciting 'student's voice'.* By posing questions with a solution-focused character the teacher can help the child begin moving towards solutions and future regarding mathematics learning. Have you ever had great math help? What did the person who gave you that do? What is your next math learning goal? What do you need to reach that goal? are examples of questions that elicit student's voice (Bannink, 2010). Also increasing waiting time after posing a question can maximise the chances of gaining insight into the child's own thinking, the child's ideas, the promotion of commitment, and increased ownership (Black et al., 2004). The proportion of total number of questions with focus on identification of child's math learning needs by actively eliciting student's voice is counted.
6. *Support given.* The teacher can provide support during a dynamic math interview. We distinguished: a) stimulating the child to write down steps in thinking, b) verbal support (e.g. hints), c) verbal support provided by notes by the teacher, d) material support (e.g. manipulate with imitation money), e) use of concrete representations of abstract models, f) use of representations of concrete mathematical actions and situations, g) clear structuring of problem/task, h) reduction of complexity, i) demonstration, and j) modelling. Support provided four times or more is indicated as most frequently provided support. The tool we developed for the conduct of a dynamic math interview contains the

aforementioned suggestions for the support that teachers can provide. Some of the suggestions have been developed by Gal'perin (1978) on the basis of Vygotksy's action theory and thus entail four levels of action: 1. informal mathematics and informal procedures; 2: representation of concrete mathematical actions and situations; 3: representation of abstract models; 4: formal mathematical operations. Other suggestions for supporting children are: the clear structuring of problems/ tasks, giving verbal hints, reducing complexity, and modelling (Van Luit, 2019). Most important is that the support be appropriate and within the child's so-called zone of proximal development.

7. *Adequate responding.* When a teacher responds to what a child says or does, they must do this in a manner which allows the child to take advantage of their response (Empson and Jacobs, 2008; Lee and Johnston-Wilder, 2013). This requires extensive mathematical knowledge for teaching (e.g. Hill et al., 2008). Adequate responding requires: insight into possible misunderstandings, provision of not only clear but also complete support, correct interpret of children's mathematical statements, determination of appropriate support and effective timing of the support. On the basis of this information, adequacy of responding can be assigned a score of 1 (= to a very small extent) to 4 (= to a very large extent).
8. *Creation of safe and stimulating climate.* Particularly for the conduct of a productive dynamic math interview, several conditions must be met: creation of a sufficiently warm and relaxed atmosphere, showing of respect, starting with a mathematical problem on which the child is likely to succeed, encouraging verbalisations, sincerity, and supportive remarks (Delfos, 2001; Ginsburg, 1997). Tell me everything you can about what you are thinking. The correctness of the answer does not matter to me. I want to know how you are trying to solve the problem. This of the dynamic math interview is assigned a score between 1 (= to a very small extent) and 4 (= to a very large extent).
9. *Teacher summary of math learning needs*. When the teacher succinctly reproduces what lies at the core of the child's needs, using the child's own words, this shows that the teacher has been listening carefully. It also allows the teacher to check their understanding of the child's math learning needs and goals. Co-responsibility on the parts of the teacher and the child is also fostered (Bannink, 2010; Delfos, 2001). Summary of math learning needs assigned a score of 0 (= not) to 1 (= to a very small extent) to 4 (= to a very large extent).
10. *Scope of the dynamic math interview.* A beneficial dynamic math interview must address various aspects of a child's mathematical development; the child's thinking and problem-solving abilities; the child's math experiences, beliefs, and emotions; and active involvement of the child in the identification what is needed for successful mathematical development (e.g., Black et al., 2004; Delfos, 2001; Ginsburg, 1997). We distinguished five types of scope, with the widest being most preferred. A teacher can focus on the child's mathematical thinking and problem-solving; the child's math experiences, beliefs, and emotions; and actively involving the child in the identification of their math learning needs (a). The teacher can focus on the child's mathematics achievement and the child's math experiences, beliefs, and emotions (b). The teacher can focus on the child's math experiences, beliefs, and emotions and the active involvement of the child in identifying their math learning needs (with no attention to mathematics achievement) (c). The teacher can focus on child's mathematics achievement and on active involvement of the child in identifying their math learning needs (with no attention to math experiences, beliefs, and emotions) (d). And finally, the teacher can focus solely on mathematics achievement (e).

Appendix D: Examples of parts of dynamic math interviews (English and Dutch)

Example in English:

A good example of actively involving the child in the identification of needs during a dynamic math interview. Below is a part of the interview.

Teacher (T): What we discovered together in this interview...on a scale of 1 to10, you assigned yourself a 5 for math when your goal was to reach a 7. Look, here we noted what we discovered [they have written down all the identified needs under the scale line drawn in the student's notebook). To reach the 7, you have to read more precisely, pay more attention during the math lesson, and join the small group that gets extra instruction. What else?

Child (C): Think for myself first.

T: Great. What else works well?

C: Paper.

T: Yes, using a notebook to organize your thinking process. What else?

C: Work precisely.

T: Yes. And you also told me that you have to read the problem thoroughly, also the title of the math problem.

C: And search for the answer.

T: For sure. Also on this problem [he points at a math problem that the child just solved], the answer was hidden, but you searched out the answer like a detective. And another point of attention was the use of a ratio table. Sometimes you used it correctly, sometimes you did not use it at all. We just solved a problem with a ratio table together and then you succeeded.

In this dynamic math interview the teacher actively involved the child in identification of his/her math needs and also wrote down specific needs under the scale line. The teacher asked questions about abilities and qualities which contributed to the child's decision to assign himself a 5 along the scale line. The teacher first addressed the child's strengths and then asked what the child needs to reach a 7. Co-responsibility for learning was promoted in such a manner.

Examples in Dutch

Voorbeeld 1: rekengesprek met een 'zwakkere' rekenaar

Leraar (L): De rekenlessen op zich in de klas, wat vind je daarvan?
Leerling (ll): Van achter kan ik het niet heel goed zien.
L: Wat bedoel je?
ll: Bijvoorbeeld links op het bord links wat daar geschreven stond met stift-pen, dat kon ik niet lezen.
L: Dus eigenlijk wil je wat dichterbij zitten dat je het wel goed kunt zien?
ll: Ja, want juf K. die wees daarnaar, maar toen dacht ik, wat staat er...want ik zie niks.
L: Heb je dat ook als ik het opschrijf?
ll: Uhm...
L: Mag je gewoon zeggen hoor.
ll: Ja, ook wel, maar dat komt omdat ik gewoon te ver van het bord af zit en dan zie ik niks.
L: Nou, duidelijk, dat betekent dat jij een plek dichterbij wil, in elk geval bij de uitleg. Dat kunnen we wel regelen. Het is belangrijk dat je het wel goed kunt zien.
ll: Ja.
L: En dan kun je daarna zelf kiezen of je daarna weer terug wil naar je eigen plekje of dat je op de plek dichterbij het bord blijft zitten.
ll: Ja.
L: Zijn er dingen die...als jij het voor het zeggen had, anders zou doen in de rekenles?
ll: Ja, uhm....ooit had ik wel iets maar ik weet niet meer wat (denkt diep na)..ik weet het niet meer, ik ben het vergeten.
L: Als het je zo te binnen schiet dan moet je het zeggen. Als jij denkt aan een rekenles, wat voor cijfer zou je een rekenles dan geven? 1 is echt verschrikkelijk en 10, dan vind je de rekenlessen super leuk. En als je dan mag kiezen (de leraar heeft schaallijn op papier getekend) wat voor cijfer zou jij de rekenlessen dan geven?
ll: Een 7.
L: Okay, een 7. Wat zou er dan moeten gebeuren in de rekenlessen zodat het een 8 of een 9 zou worden?
ll: Dat ik beter zelf kan rekenen en alles.
L: Wat zou je kunnen helpen om beter zelf te kunnen rekenen?
ll: Dat ik minder stress in mijn hoofd heb.
L: Hoe zouden we die stress weg kunnen krijgen? Je hebt in elk geval al aangegeven dat je een plek dichterbij het bord nodig hebt om het goed te kunnen zien, maar wat nog meer?
ll: een stressbal

Et cetera. bij deze leerling bleek stress bij rekenen een belemmerende factor te zijn, vooral bij toetsen. Met de leerling is hierna doorgepraat over waar de stress vandaan komt. Hij vindt het moeilijk om zich lang te concentreren en ervaart ook stress door zich te meten met anderen. In samenspraak met de leerling zijn diverse suggesties naar voren gebracht en het gesprek is afgesloten met duidelijke afspraken met commitment van de leerling.

Voorbeeld 2: rekengesprek met een 'gemiddelde' rekenaar (twee fragmenten uit een rekengesprek)

L: Heb je ook wel eens een som gemaakt waarvan je eerst dacht: wat moet ik hiermee…ik snap er niets van en waarbij het uiteindelijk toch is gelukt om de som te maken?

ll: Ja.

L: En hoe heb je dat toen aangepakt?

ll: Ik ging eerst even goed kijken naar die som en toen ging ik die in mijn rekenschrift uitrekenen. Alleen, ik snapte het toen niet en toen keek ik nog een keer goed naar de uitleg en keek nog eens goed wat er allemaal precies stond. Dat moest ik wel een paar keer doorlezen, maar toen dacht ik: Oh ja, zo moest het!

….

Opgave betreft vier ijsjes, waarbij de prijs per bolletje moet worden vergeleken. De leerling is dit aan het uitrekenen voor een ijsje met vier bolletjes dat € 2,40 kost.

ll: Ik maak er een makkelijke van, doe ik 24 gedeeld door 4 is uhm 6, want 6 x 4 is 24. Uhm, ik heb er een nul afgehaald, dus dan moet ik er nog een nul bij doen, dus dat is 60.

L: Ja, en wat is die 60 dan?

ll: 60 cent voor 1 bolletje. Dus dan is die nog steeds meer dan die (ander ijsje), dus antwoord c moet het zijn, die prijs is het laagst.

L: Heel goed, dat heb je goed doordacht. Ik begrijp nu helemaal hoe je achter het antwoord bent gekomen.

Voorbeeld 3: gesprek met een wat 'sterkere' rekenaar (weergave van een fragment middenin het rekengesprek………)

ll: Ja, meestal probeer ik wel mijn hersens te kraken.
L: (knikt bevestigend). Hoe voelt dat dan als je zo'n opgave, waarbij je echt je hersens moest laten kraken, toch hebt opgelost?
ll: Dan voel ik me eigenlijk best gelukkig.
L: Ja. Gaaf joh. Dat is een heel mooie eigenschap dat je dan gewoon doorzet om het te blijven proberen. Super. We gaan eens kijken welke sommen al super goed gingen en sommen die je nog wat lastig vond. En ik ben heel benieuwd hoe je het dan uitrekent. Misschien ontdek je dan zelf wel hoe je het hebt gedaan en wat er is mis gegaan. Nou, dan gaan we eerst beginnen met eentje die je lastig vond. Hier heb je een kladblaadje, want dat zeg ik ook altijd: Je mag altijd een kladblaadje gebruiken. Lees de opgave over het varken eens voor.
ll: (leest opgave voor)…dit stuk marsepein weegt 1 kilo. J. koopt de helft. Hoeveel kost dat stuk?
L: Nou, vertel eens hardop hoe je dit gaat aanpakken.
ll: (Denkt zichtbaar na) en zegt: Nou, dat weet ik eigenlijk niet, want volgens mij is 100 gram geen 1 kilo.
L: Nee. Dus?
ll: Moet je dan eerst keer 10 doen?
L: Waarom zou je dat doen?
ll: Nou, dan is het 1 kilo.
L: Ah, dat klinkt goed. Doe maar.
ll: Dat is het 12 euro, en dan door de helft is 6 euro.
L: Je had eerder als antwoord gegeven: 60 cent. Wat heb je gedaan, denk je?
ll: Ja, ik dacht meteen door de helft en dat heb ik toen gedaan.
L: Dacht je toen dat 100 gram hetzelfde is als 1 kilo?
ll: Ja, ik ging toen een beetje te snel en dacht er niet goed over na.

L: Ah, want je kunt het heel duidelijk vertellen: dit klopt nog niet, want dat is 100 gram en dat is niet 1 kilo. Dus je bent te snel gegaan, je dacht door de helft en klaar.
ll: Ja.
L: Wat zou je kunnen helpen bij deze som?
ll: Eerst goed kijken.
....
(wat verderop in het rekengesprek)
L: En dan hebben we er nog eentje. Even kijken, eentje die je wat lastiger vond, met de tennisballen. Lees eens voor.
ll:. In een koker gaan 3 tennisballen, hoeveel tennisballen gaan er in totaal in deze doos?
(ll denkt zichtbaar na.)
L: Leg eens uit, hardop. Vertel hardop wat je aan het doen bent.
ll: Ja. Toen dacht ik, ik doe daar nog allemaal van die kokers. Uhm..alleen ik had beter.... en toen werd het het foute antwoord, ik had beter gewoon 3 keer 4 kunnen doen.
L: Kijk, want jij zei het zijn 24 kokers. Dus wat heb jij gedaan tijdens deze opdracht?
ll: (denkt zichtbaar na.) Ja, ik dacht 24, uhm, tennisballen, uhm, wacht ben het even kwijt.
L: Maakt helemaal niet uit, rustig aan, en anders lees je de som nog een keertje.
ll: In een koker gaan drie tennisballen, hoeveel tennisballen gaan in totaal in deze doos? (ll denkt zichtbaar na). Nou, ja ik wist wel dat het ging om de ballen, alleen toen had ik het per ongeluk keer 4 gedaan.
L: Keer 4 gedaan. En doe je dit allemaal uit je hoofd?
ll: Nou, bij deze moest ik echt wel even mijn hersens kraken, alleen, ja uiteindelijk ga ik dan ook het kladblaadje gebruiken.
L: Nou, hij ligt er. Dus probeer hem nu eens op te lossen.
ll: (schrijft, werkt opgave uit). Gewoon, 3 keer 4.
L: Waarom 3 keer 4?
ll: Omdat het drie kokers zijn, hier vier kokers.
L: Ah, dus je ziet meteen dat je dan niet de lege plekjes hoeft te tellen.
ll: Ja. (ll schrijft verder)...is 12.
L: Ja.
ll: En weer 12 keer 3 (ll werkt verder uit).
L: En waarom moet dan nog 12 keer 3?
ll: Omdat in elke koker 3 tennisballen gaan en ze willen weten hoeveel tennisballen.
L: Oh oké. Dus ze willen niet weten hoeveel kokers?
ll: En dan is het 36. (ll is aan het schrijven-uitwerken).
L: Ja. Heel goed. Dus wat denk je dat er eerder mis was gegaan?
ll: Ja dat ik het daar ging opvullen, terwijl ik het verkeerd had opgevuld.
L: Oh ja, je had te weinig kokers gedaan, of te veel?
ll: Volgens mij te veel.
L: Maar jij zei 24 ballen, dus ik denk dan te weinig.
ll: Ja waarschijnlijk dacht ik, uhm... 8 kokers.
L: Ja. Dat kan. Uhm, maar je ziet het eigenlijk meteen al.
ll: (glimlacht). Ja.
L: Wat er mis is gegaan? Want je kijkt even, en dan vertel je, oh, ik heb dit gedaan, en dat klopt precies. Je zei: ik had gewoon 4 keer 3 moeten doen. Dat is hartstikke knap dat jij dat ziet. Waar kun je nu op letten bij het maken van dit soort opgaven?
ll: Ja, gewoon, wat beter kijken, eh, naar wat je uit moet rekenen en hoe.
L: (knikt bevestigend). En wat helpt jou daarbij?
ll: (ll denkt zichtbaar na). Nou, eigenlijk gewoon wat meer tijd nemen.
L: (knikt bevestigend). Soms nog te snel?
ll: Ja.
L: Dat is mooi als je wat meer tijd neemt. Is er nog iets anders wat je nodig hebt op het gebied van rekenen?
....

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